DELTA HEDGING: FUTURES VERSUS UNDERLYING SPOT

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Abstract

Derivative exchanges and the trading of derivatives have existed for a long time. CBOE[1] started trading call options way back in 1973. In South Africa, an organised trading in index future contracts started with the establishment of Safex[2] during 1988. Trading in single name futures (also known as single stock futures) started in 1997. The liquidity in the ALSI and DTOP index futures as well as the liquidity in most of the FTSE/JSE TOP 40 single name futures has grown tremendously over the past 5 years. A liquid and vibrant onscreen market makes it now possible to use these futures as alternative hedging instruments.

This short note explains how to use a futures contract to hedge a position in the underlying spot contract or vice versa e.g., ALSI futures vs. shares or an MTN future versus MTN shares or a Rand/Dollar future versus a spot FX contract. We will also discuss a simple conversion measure that can be useful if one wants to hedge a position in a certain futures contract with another related futures contract or another related spot contract e.g., hedging a DTOP or INDI futures position with an ALSI futures contract or FTSE/JSE TOP 40 index[3] spot position (or shares).

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[1]Chicago Board Options Exchange — www.cboe.com

[2]The South African Futures Exchange

[3]The FTSE/JSE Top 40 Index consists of the largest 40 companies ranked by full market value. For a full description surf to www.ftse.com/Indices/FTSE_JSE_Africa_Index_Series/index.jsp

1 Introduction: the Basics

In this document we always assume that, due to no arbitrage, the value of a futures contract is equivalent to its fair value or forward price [Hu 06] i.e.,

$$F = S\, e^{(r-d)t} \tag{1}$$

where
S is the spot/cash value of the underlying asset
F is the fair value or future on the underlying asset S
r is the risk-free interest rate in continuous format
d is the underlying asset's dividend yield in continuous format
t is the fractional time till the expiry of the futures contract
$r - d$ is known as the "cost of carry".

2 From Notional Value to the Number of Contracts

If we have a cash amount and we want to invest that into a South African index asset, we need to determine how many contracts, C, we need to buy to get the required exposure.

This is done through

$$C = \frac{A}{S * M} \tag{2}$$

where A is the cash amount in Rand, S is the spot/cash value of the index and we have a scaling factor (also called the index multiplier) denoted by M. In South Africa, index contracts trade at 10 times the index [Ko 02] — this is equivalent to each index point carrying a value of R10 – and $M = 10$. The Rand/Dollar currency future has a multiplier of $M = 1000$ because we have $1000 per contract.

Similar, if we want to invest the cash into futures we have

$$C = \frac{A}{F * M} \tag{3}$$

with F the index futures value, defined in (1).

As an example, the March 2011 ALSI future is currently trading at 28,192. If an investor wants to invest R100 million, he needs to buy 374.71 March 2011 ALSI contracts. This number will be rounded to either 375 or 374 according to the investor's preferences.

3 Delta Hedging using Futures

In practice, hedging is often carried out using a position in futures rather than one in the underlying cash asset. Mathematically, under the no-arbitrage argument, it can be shown that $e^{(r-d)t}$ futures contracts have the same sensitivity to stock price movements as one cash contract.

Let's explore the last statement and use some option terminology. Assume we have a long position in an underlying index, like the TOP 40, that has a current value of S_0. We now want to hedge this position with a futures contract (short the future) such that if the TOP 40 index changes to S_1, we do not loose or make any money – we are delta-neutral. We thus need as many futures such that

$$N_S\ (S_1 - S_0) - N_F\ (F_1 - F_0) = 0 \tag{4}$$

on a value for value basis where N_S is the number of cash contracts and N_F is the number of futures contracts.

Now note, from equation (1) that when the index value changes by ΔS, the futures price changes by

$$\Delta F = \Delta S\, e^{(r-d)t}. \tag{5}$$

This leads us to the definition of the delta of a futures contract to be

$$\Delta_F = \frac{\Delta F}{\Delta S} = e^{(r-d)t}$$

From this we ascertain that if S changes, the futures level changes Δ_F times more. However, we need (4) to be valid. To counter the influence of the futures delta in (5), we need to adjust our positions. If our cash position is given by N_S we need N_F futures contracts to hedge this given by

$$N_F = N_S\, e^{-(r-d)t} \tag{6}$$

On the other hand, if we have a position in a futures contract and we want to hedge using the spot contract we use

$$\begin{aligned} N_S &= N_F\, e^{(r-d)t} \\ &= N_F\, \Delta_F \end{aligned} \tag{7}$$

4 Hedging a Spot Contract with a Different Spot Contract

If we have a position in one asset and we want to hedge that using a different asset (for instance, hedging the DTOP index with ALSI stock), we do the following: we assume there is a correlation of 1 between the two contracts and we do a value for value conversion where, from Eq. (2), the number of ALSI contracts is given by

$$C_A^U = \frac{A}{S_A * M}$$

with S_A the ALSI spot value, C_A^U is the number of ALSI spot contracts (in stock) needed and M the multipliers. We also have the number of DTOP spot contracts given by

$$C_D^U = \frac{A}{S_D * M}$$

with S_D the DTOP spot value. Due to the common notional value A, we can manipulate the two equations to give

$$C_A^U = C_D^U \frac{S_D}{S_A} \tag{8}$$

Equation (8) thus gives us the number of ALSI contracts we need to buy/sell to hedge a position in the DTOP index. The number of DTOP contracts is thus simply multiplied by the ratio between the two spot values.

This simplistic hedging procedure is only viable if the two instruments has a correlation close to one.

5 Hedging a Futures Contract with a Different Futures Contract

If we have a position in the SWIX/DTOP future and we want to hedge that using ALSI futures we use Eq. (3) and, similar to the previous section, obtain

$$C_A^F = C_D^F \frac{F_D}{F_A} \tag{9}$$

with F_D the SWIX/DTOP futures value and F_A the ALSI futures value and, C_A^F is the number of ALSI futures contracts and C_D^F is the number of Swix/DTOP futures contracts. However, due to no-arbitrage arguments and Eq. (1) we have

$$\begin{aligned} C_A^F &= C_D^F \frac{S_D\, e^{(r-d_D)t_D}}{S_A e^{(r-d_A)t_A}} \\ &= C_D^F \frac{S_D}{S_A} e^{(d_A t_A - d_D t_D)} \end{aligned} \tag{10}$$

where S_D is the SWIX's/DTOP's spot value. Also

t_D is the time till the DTOP future expires;

t_A is the time till the ALSI future expires;

d_D is the DTOP's dividend yield; and

d_A is the ALSI's dividend yield.

Equation (10) is thus similar to Eq. (8) with a dividend yield cost factor added. If we now assume that the two futures expires on the same date we have

$$C_A^F = C_D^F \frac{S_D}{S_A} e^{(d_A - d_D)t} \tag{11}$$

If we further assume that the two dividend yields are the same, Eq. (9) reduces to

$$C_A^F = C_D^F \frac{S_D}{S_A} \tag{12}$$

6 Hedging Futures using a Spot Contract

If we have a position in a futures contract and we want to hedge that using a different, related, futures contract, we use Eq. (10) to obtain the number of contracts we need to buy/sell. If we now want to go one step further and hedge that futures position (ALSI) using the spot underlying (ALSI shares) we have from (7) and (10)

$$\begin{aligned} C_A^U &= C_A^F e^{(r-d_A)t_A} \\ &= C_D^F \frac{S_D}{S_A} e^{(rt_A - d_D t_D)}. \end{aligned} \tag{13}$$

Again, if we assume that the two futures contracts expires on the same date we have

$$C_A^U = C_D^F \frac{S_D}{S_A} e^{(r-d_D)t}. \tag{14}$$

7 Delta Hedging and Trading Systems

Any derivatives trading and risk management system (for equities, foreign exchange and bond derivatives) should accommodate conversions as described above. The following delta hedging conversions should be included

- Hedging a future by its underlying stock or vice versa — e.g., hedging an ALSI futures position with the underlying ALSI shares. These conversions are done by applying Eqs. (6) or (7).
- A position in one underlying can be hedged with another underlying (e.g., a DTOP position hedged with ALSI stock). Conversion done by applying Eq. (8).
- A position in one futures contract can be hedged in another futures contract. The conversion is done by applying Eq. (11).
- A position in a futures contract can be hedged with another spot asset (e.g., hedging a DTOP futures position with ALSI stock). This conversion is obtained by applying Eq. (14) where we always assume that the two futures contracts expire on the same date.

References

[Hu 06] J. Hull, *Options, Futures, and other Derivatives*, 6^{th} Edition, Pearson International Edition (2006)

[Ko 02] A. A. Kotzé, *Equity Derivatives: effective and practical techniques for mastering and trading equity derivatives*, Workshop proceedings (2002)