Black-Scholes or Black Holes?

FINANCIAL CHAOS THEORY

P.O. Box 16185
Doornfontein, 2028
South Africa

Tel: (+27 11) 402 3170
Fax: (+27 11) 402 6920
consultant@quantonline.co.za
www.quantonline.co.za

The Nobel laureates Fischer Black, Myron Scholes and Robert Merton revolutionised financial economics with the publication of their option valuation formula in 1973. The model, however, was devised for an elementary, ideal and frictionless world. Understanding the framework and underlying simplifying assumptions behind their formulation will help to minimize the risks in trading and managing derivative securities.

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The Brady Commission that investigated the 1987 crash on Wall Street put part of the blame on the trading strategies derived from the Black-Scholes (BS) analysis. In 1994 George Soros reiterated that, if there is an overwhelming amount of delta hedging in the same direction, the theoretical equilibrium will be disturbed leading to discontinuous price movements - the premise of the BS model will break down. A well-known example is the downfall of Long Term Capital Management (LTCM) in 1998. In a recent letter to shareholders Warren Buffett warned that derivatives were “financial weapons of mass destruction” and derivatives trading sat on a “time bomb”.

Saggitarius A*: supermassive black hole at the Milky Way’s center

Statements like these contain some truths. It is due to many practitioners seeing the formula as a *black hole* of gravitating mathematical forces that overwhelms them. They perceive the model as flawed but, contrary to belief, the flaws lie in misconceptions and its practical application. Many traders, fund and risk managers use the outputs clinically in their analysis. This “black box” approach is wrong. It is imperative that every derivatives practitioner understands the universe in which the BS formula was devised. In this note we discuss and illuminate the *black hole* surrounding this enigmatic formula.

Mathematical modelling of financial markets can be traced back to Louis Bachelier’s 1900 dissertation on speculation in the Paris markets. Financial economics, however, only came of age in 1973 with the publication of the preference-free option pricing formula by Fischer Black, Myron Scholes and Robert Merton. Their model established the everyday use of mathematical models as essential tools in the world of finance, both in the classroom and on the trading floor.

One fact we constantly have to remember is that the BS model is just a model, an abstraction of reality. Modelling, however, is not merely a collection of techniques but an art in blending the relevant aspects of a problem and its unforeseen consequences with a descriptive, yet tractable, mathematical methodology. Building and analysing models are important. This is illustrated by the following benefits that resulted from the BS formula.

- the concepts behind the formula provided the framework for thinking about option valuation and dynamics;
- such research led to an expanded universe of financial instruments encompassing more complex option structures;
- quantitative risk management, stress testing and scenario analysis became possible;
- hedging of derivatives were easier.

You might feel that quantitative analysts confuse everyone with their mathematical jargon and, shouldn’t the market determine the prices of traded securities? That is correct and hence we talk about option *valuation* or *estimation*. *Pricing* generally refers to where the market price is rather than where it should be.

One of the first lessons in option valuation is that the model should only be used to guide us toward the correct *value,* giving somebody a BS calculator doesn’t make him a trader. The model is a means of thinking about the problem we’re tackling *but it does not necessarily give us the answer*. It is a filter that lets us turn our perceptions about volatility into Rand values or hedge ratios. Traders use the model as a standardized communication tool for productive trading conversations.

How should the BS model then be used? A good start is to comprehend its limitations and applicability to the case at hand. By considering practicalities like transaction costs, liquidity and frequency of hedging, we will be guided on how to
overlay these problems onto the actual bare-bones price of the model and decide whether that's a price we can live with. Trading with a model is definitely not the simple and clinical procedure many people imagine.

Let's now look at the BS world, meaning the environment they created which underpins their formula. They assumed:

- *the underlying stock price follows a continuous random walk* – stock prices diffuse through time linearly proportional to the spot price i.e. constant volatility. The price in the future is unpredictable but will most likely be close to some mean or expected value,
- *the efficient market hypothesis holds* - markets are liquid, have price-continuity, are fair, are complete and all players have equal access to information,
- *investors live in a risk-neutral world* - they require no compensation for taking risk. There are no arbitrage opportunities and the expected return is the risk-free interest rate. This was perhaps their most important insight leading them to construct a self-financing riskless hedge; in a portfolio of three securities - an option, the underlying stock and a riskless money market security - any two could be used to exactly replicate the third by a trading strategy,
- *delta hedging is done continuously* - for the return on the hedge portfolio to remain riskless, the portfolio must continuously be adjusted as the asset price changes over time.

In the rest of this note we will look at the two most common violations of the BS environment namely imperfect volatility forecasts and dynamic hedging.

We first look at dynamic hedging. In the BS environment, the key to understanding derivatives is the notion of a premium. The premium of an option is given by the model with the volatility as a vital input, but the premium and the traded price of an option are rarely the same. This is explained by the concept of delta-hedging. The delta tells a trader to buy or sell the underlying stock in order to hedge the market risk of an option. In theory, if delta-hedging is done dynamically, trading profits (losses) should equal premium outflows (income).
The premium is thus equivalent to the cost of hedging the option. However, dynamic hedging in real markets is not a risk-free proposition. Some of the additional risks are changes in volatility, changes in interest rates, changes in dividends, trading costs and liquidity. Risk is reduced if we know what our real hedge strategy and portfolio is and what it is going to cost. This is not realistic but it does mean that the true value of the option, in general, is substantially different to the BS value.

Secondly we look at the concept of volatility. The problem arises because we should use the future volatility in the model, which is not known and needs to be estimated. Many practitioners use the implied volatility in managing their books. Implied volatility is seen as the market's collective forecast of volatility to be realised over the life of the option. Studies comparing implied volatilities with actual realised volatilities generally find little agreement between the two. As far back as 1972, BS noted that the implied volatility employed by the market is too narrow and that the historical estimates of volatility include an attenuation bias.
Today, this is called the volatility skew or term structure of volatility. The skew is a consequence of the market's view that options with different strikes and different expiries, have different risks and should be valued as such. It only rose to prominence after the crash of October 1987 reflecting a risk or "crash" premium.

Using implied volatility is complicated because it might include the effect of mark-ups and it is impossible to derive any volatility expectation from a single option contract – a problem in illiquid markets. Because of these problems some traders use implied volatility for valuations but their own forecasts (utilising Garch analysis for instance) for hedging.

This attribution belies the BS picture where it is assumed that there is only one underlying stock and hence one evolutionary process. The skew and different volatilities used for the same trade show that the real underlying process differs from the random walk assumed by BS.

To overcome this, researchers look at different processes like the parabolic, Levy or jump diffusion processes. Due to its complexity this has not caught on but models where the volatility is assumed to follow a stochastic process are being used successfully. However, many skilled traders still use BS and they account for this “discrepancy” by utilising the volatility skew effectively, that is, volatility is used to reflect all the un-tradable risks in the market.

From the abovementioned arguments we see that the BS assumptions are demonstrably wrong for real world markets. Still the model is used by everyone working in derivatives. It is used confidently in situations for which it was not designed for, usually successfully and the model is remarkably robust.

The black hole is not that dark and practitioners who understand the BS environment use a “bag of tricks” to map this simplified world to the real one. They use stress testing, scenario analysis and procedures like Monte Carlo simulations where they can calculate option values and hedge ratios discretely in time. They also learn to use fudges like shadow delta and shadow gamma.

Intelligent traders iterate between imagination and model use in a way that belies easy categorization and testing. There is an old saying, “It isn’t the size of the wand; it’s the wizard who waves it”. This is precisely the reason why the BS model has survived all these years. It does not try to be fancy, it lets the operator himself be fancy at will!

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