Exotic Options and their Valuation Methodologies

All options will be European in nature.

Forward Start Option Options that are paid for in the present with a strike price determined at a subsequent date prior to the expiry date. Payoff = $\max\left[0, \phi\left(S(T) - \alpha\, S(t_1)\right)\right]$	Can be valued using a closed-form analytic formula within a Black-Scholes framework [Hu 02]. Forward interest rates and forward volatilities will be used in the valuation.
Asian Option (fixed strike) An option where the payoff is determined by the arithmetic average underlying price over some pre-set period of time. Payoff = $\max\left[0, \phi\left(\frac{1}{n}\sum_{i=1}^{n} S(t_i) - K\right)\right]$	No analytic formula exists. We will use Curran's approximation that adheres to all Black-Scholes principles. This method gives good accuracy and is easy to implement where the average is taken on certain pre-specified dates [Cu 94; Ni 01].
Barrier Options (single barrier) Vanilla barrier options are path-dependent with the key characteristic that they are either initiated or exterminated when the underlying spot price reaches a certain pre-specified level (barrier); that is, they are either knocked in or knocked out. Types: 1. Up & In – knock-in 2. Up & Out – knock-out 3. Down & In – knock-in 4. Down & Out – knock-out	Can be valued using a closed-form analytic formula within a Black-Scholes framework [Hu 02; GD 07; Ha 07] The aforementioned analytic formulas present a method to price barrier options in continuous time. However, most of the time, prices are sampled at discrete times. For this we use the Broadie, Glasserman & Kou continuity correction [BG 97] for daily, weekly or monthly monitoring.
Digital/Binary Options A binary option is an option where the payoff is binary: it is either some fixed amount of cash/asset or nothing at all. Types: 1. Cash-or-nothing: pays a pre-defined fixed amount of cash if in-the-money. 2. Asset-or-nothing: pays the value of the underlying security in cash if in-the-money.	Options in continuous time can be valued by analytic formulas within the Black-Scholes framework [Hu 02; GD 07, Ha 07].

Lookback Options (fixed/floating strike)

Lookback options are a type of path-dependent option where the payoff is dependent on the maximum or minimum asset price over a part of the life of the option.

Payoff(floating strike call) = $\max\left[0, S(T) - \min\left[S(t_0, T)\right]\right]$

Payoff(floating strike put) = $\max\left[0, \max\left[S(t_0, T)\right] - S(T)\right]$

Options in continuous time can be valued by an analytic formula within the Black-Scholes framework [GD 07]. Monte Carlo simulation will be used if prices are sampled at discrete times.

Payoff(fixed strike call) = $\max\left[0, \max\left[S(t_0, T)\right] - K\right]$

Payoff(fixed strike put) = $\max\left[0, \min\left[S(t_0, T)\right] - K\right]$

Cliquet/Ratchet Options

A Cliquet option (or ratchet option) settles periodically and resets the strike at a factor of the then spot level. Gains are locked in. It is, therefore, a series of vanilla options but, where the total premium is determined in advance.

Types:

1. Pay-at-end: payout is paid at final expiry.

Payoff = $\sum_{i=1}^{n} \max\left[0, \phi\left(S(t_i) - \alpha\, S(t_{i-1})\right)\right] e^{r_{i,n} t_{i,n}}$

2. Pay-as-go: payout paid at each reset date.

Payoff = $\sum_{i=1}^{n} \max\left[0, \phi\left(S(t_i) - \alpha\, S(t_{i-1})\right)\right]$

The Cliquet option starts out as an ordinary vanilla option with a fixed strike price, but the strike is reset to the market price times a factor on a set of predetermined dates. When the strike price is reset, any positive intrinsic value is locked in.

A Cliquet option is thus a vanilla option together with a series of forward start options. Analytic formulas exist and will be used.

A relevant yield curve will be used to calculate forward rates and a volatility surface will be used to obtain forward volatilities.

Symbol Definition

$S(t)$ is the underlying's spot price at time t. K is the strike price. $\phi = 1$ for a call, -1 for a put.

T is the time from valuation till expiry. t_0 is the time of valuation - usually zero.

t_1 is any time between t_0 and T. t_i is any time from t_0 to T. $t_{i,n}$ is the time from t_i to t_n.

$r_{i,n}$ is a continuous interest rate for the period from t_i to t_n where $t_n = T$.

α is a factor specifying how far in-the-money or out-the-money an option is.

n specifies the number of dates in a Cliquet or Asian option.

References

[Hu 02] Hull, J., "Options, Futures & Other Derivatives", 5th Edition, (2002)

[Ni 01] Nielsen, L., "Pricing Asian Options", Masters thesis (2001)

[Cu 94] Curran, M., "Valuing Asian and Portfolio Options by Conditioning on the Geometric mean Price", Management Science, 40, (1994)

[GD 07] http://www.global-derivatives.com

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[BG 97] Broadie, M., Glasserman, P., Kou, S.G., "A Continuity Correction for Discrete Barrier Options", Journal of Mathematical Finance, (Oct '97)

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