JOHANNESBURG STOCK EXCHANGE
Equity Derivatives

Variance Futures: Initial Margin

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Introduction

Variance Futures are contracts that obligate the holder to buy or sell variance at a predetermined variance strike at a specified future time. Variance has the interesting property of directly increasing with volatility. Hence, a direct exposure to volatility is therefore afforded by having a position in variance futures.

Volatility

Volatility is a measure of risk or uncertainty and it has an important role in the financial markets. Volatility is defined as the variation of an asset's returns – it indicates the range of a return's movement. It can be said to measure the overall level of changes in the asset prices. A position in a variance futures contract therefore provides exposure to the overall changes in the underlying asset prices. Variance future prices are independent of the direction of the underlying asset prices[3].

Payoff and Initial Margin

Consider a variance future with a variance strike K. The payoff of one variance futures contract at a future date T (the variance future expiry date), is

$$Payoff_T = NumberContracts \times VPV \times [\text{Realised Variance} - K]$$

with VPV the value of 1 variance point. The initial margin (1-day value at risk measure) of a variance future, is given by

$$IMR = NumberContracts \times VPV \times [2\lambda\sqrt{K} + \lambda^2]$$

The parameter λ is calculated as the maximum change over the historical variance future strike values. It measures the expected one day volatility of volatility inferred from the historical volatility skews and varies per variance future expiry.

The number of variance future contracts to buy/sell for a certain volatility ZAR exposure, Vega[4], is given by

[1] From Financial Chaos Theory: surf to www.quantonline.co.za
[2] From the JSE and Safex
[3] Equivalently, Variance Futures are delta neutral instruments.
[4] The average exposure to a 1 point move in volatility, is given by the Vega, for a Variance Future

$$NumberContracts = \frac{Vega}{2\sqrt{K}}$$

Example: Suppose a market participant wants to hedge a 6-month volatility exposure of R500 000 (Vega). The participant decides to go long a standard[5] variance future contract that expires in 6-months time. Let's assume the current volatility is 20% and that the variance future is struck at a variance strike of 20^2 (K). We further ascertain that the current 6-month variability of volatility parameter λ is 3%.

A long volatility exposure, Vega of R500 000, is equivalent to going long

$$NumberContracts = \frac{Vega}{2\sqrt{K}} = \frac{500000}{2\sqrt{20^2}} = 12{,}500 \text{ Variance Future Contracts}$$

The total initial margin requirement for going long 12,500 of these Variance Future contracts, is

$$IMR = NumberContracts \times VPV \times [2\lambda\sqrt{K} + \lambda^2]$$

$$IMR = 12500 \times 1 \times [2(3)20 + 3^2] = 1{,}612{,}500$$

$$IMR / Vega = \frac{1{,}612{,}500}{500{,}000} = 3.225$$

Equivalently the

$$IMR = \frac{1{,}612{,}500}{500{,}000} = 3.225 Vega$$

Thus the initial margin requirement covers the holder for a 3 point possible daily move in the volatility.

Suppose after 6-months volatility is higher, and the realized volatility is now 25. The payoff to the long holder of the 6-month Variance Future is then

$$Payoff_T = Contracts \times VPV \times [\mathrm{Re}alisedVariance - K]$$

$$Payoff_T = 12500 \times 1 \times [25^2 - 20^2] = 2{,}812{,}500 ZAR = 5.625 Vega$$

If this variance future was sold over-the-counter, (usually as a variance swap), the price of the equivalent variance swap would be

$$= NumberContracts \times K$$
$$= 12500 \times 20^2 = 5{,}000{,}000\, ZAR = 10\ Vega$$

IMR for the 3 and 6 month Contracts

The initial margin requirements (IMR) of variance futures that has already started, i.e. variance futures that already accrued realized variance, are determined as the initial margin of a new variance future (i.e. Variance Future with no accrued realized variance),

[5] Standard Variance Future contracts has a variance point value of 1ZAR

discounted using a time-weighted discount factor. The time-weighted discounting is introduced to account for the fact that the elapsed variance future already accrued realised variance. Table 1 and Table 2 give the initial margin requirement discounts, applicable to the 3-month and 6-month variance futures.

Time Elapsed Since Initiation of 3month Variance Future		**Margin as % of** IMR
Month	0 to 1	100%
Month	1 to 2	67%
Month	2 to 3	33%

Table 1: Reduction in initial margin requirements for a 3 month variance future.

Time Elapsed Since Initiation of 6-month Variance Future		**Margin as % of** IMR
Month	0 to 1	100%
Month	1 to 2	83%
Month	2 to 3	67%
Month	3 to 4	50%
Month	4 to 5	33%
Month	5 to 6	17%

Table 2: Reduction in initial margin requirements for a 6 month variance future.