

Variance Futures listed on Safex: a short note V1.0

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1. Introduction

Volatility is a measure of the risk or uncertainty and it has an important role in the financial markets. Volatility is defined as the variation of an asset's returns – it indicates the range of a return's movement. Large values of volatility mean that returns fluctuate in a wide range – in statistical terms, the standard deviation is such a measure and offers an indication of the dispersion or spread of the data².

Volatility forms an integral part of the Black-Scholes-Merton option pricing model [BS 73]. Even though the model treats the volatility as a constant over the life of the contract, these three pioneers knew that volatility changes over time. As far back as 1976, *Fischer Black* wrote [BI 76]

"Suppose we use the standard deviation ... of possible future returns on a stock ... as a measure of volatility. Is it reasonable to take that volatility as constant over time? I think not."

Black&Scholes defined volatility as the standard deviation because it measures the variability in the returns of the underlying asset [BS 72]. They determined the historical volatility and used that as a proxy for the expected or implied volatility in the future. Since then the study of implied volatility has become a central preoccupation for both academics and practitioners [Ga 06].

Figure 1 shows a plot of the 3 month historical volatility for the JSE/FTSE Top 40 index since January 2007 using daily data. It is clear that volatility is not constant.

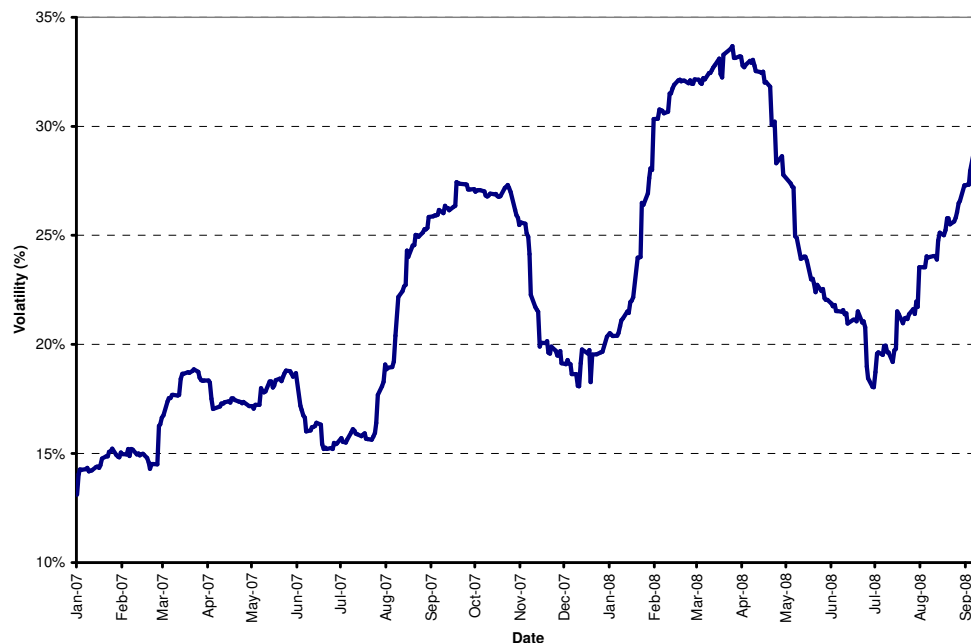


Figure 1: Three month historical rolling volatility for the JSE/FTSE Top 40 index from January 2007 to September 2008.

² The standard deviation is one of the fundamental elements of the Gauss or normal distribution curve – it describes the width of the famous bell curve.

Volatility is, however, statistically persistent, i.e., if it is volatile today, it should continue to be volatile tomorrow. This is also known as volatility clustering and can be seen in Figure 2. This is a plot of the logarithmic returns of the Top 40 index since June 1995 using daily data.

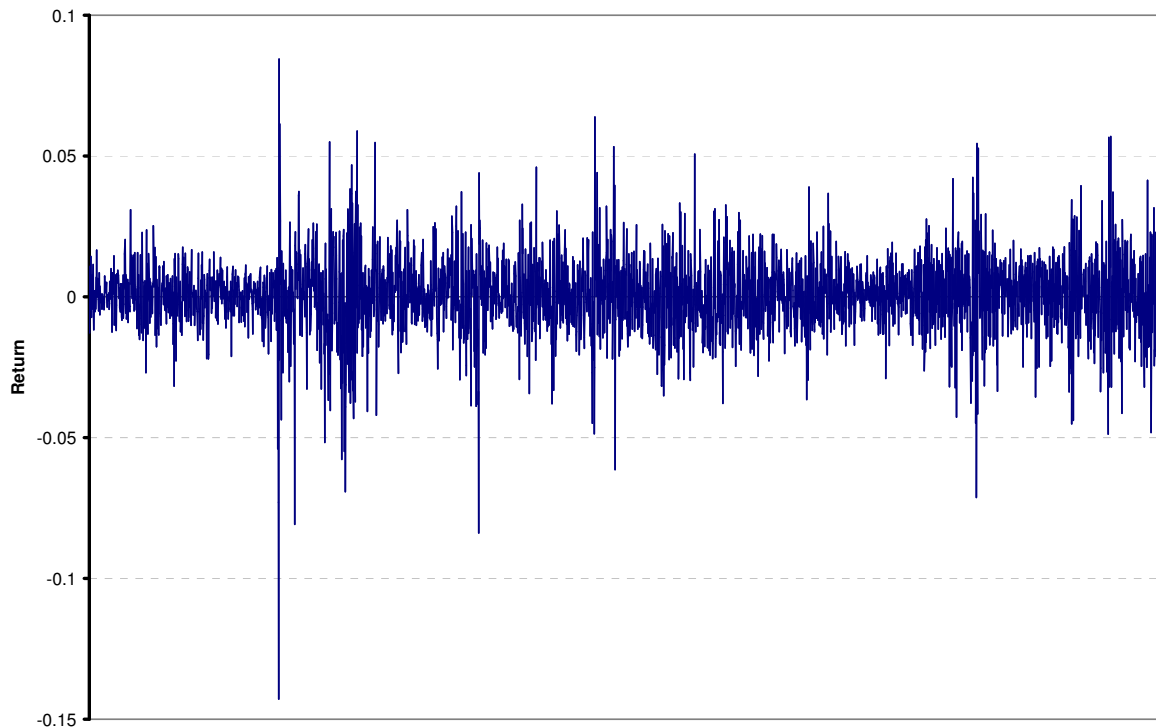


Figure 2: JSE/FTSE Top 40 daily log returns from June 1995 to September 2008. Note the 13.3% negative return on 28 October 1997 – start of the Asian crises.

But, what is this “volatility”? As a concept, volatility seems to be simple and intuitive. Even so, volatility is both the boon and bane of all traders --- you can't live with it and you can't really trade without it. Without volatility, no trader can make money! Most of us usually think of “choppy” markets and wide price swings when the topic of volatility arises. These basic concepts are accurate, but they also lack nuance.

How do we define the volatility (standard deviation) as being good, or acceptable, or normal? By many standards, a large standard deviation indicates a non-desirable dispersion of the data, or a wide (wild) spread. It is said that such a phenomenon is very volatile – such phenomena are much harder to analyze, or define, or control. Volatility also has many subtleties that make it challenging to analyze and implement [Ne 97]. The following questions immediately come to mind: is volatility a simple intuitive concept or is it complex in nature, what causes volatility, how do we estimate volatility and can it be managed?

The management of volatility is currently a topical issue. Due to this, many institutional and individual investors have shown an increased interest in volatility as an investment vehicle.

2. Volatility as an Investable Asset Class

Apart from its prominent role as financial risk measure, volatility has now become established as an asset class of its own. Initially, the main motive for trading volatility was to manage the risk in option positions and to control the Vega exposure independently of the position's delta and gamma. With the growth of the volatility trading segment, other market participants became aware of volatility as an investment vehicle [HW 07]. The reason is that volatility has peculiar dynamics:

- It increases when uncertainty increases.
- Volatility is mean reverting - high volatilities eventually decrease and low ones will likely rise to some long term mean.
- Volatility is often negatively correlated to the stock or index level.
- Volatility clusters.

How can investors get exposure to volatility? Traditionally, this could only be done by taking positions in options, and then, by delta hedging the market risk. Delta hedging, however, is at best, inaccurate due to the Black&Scholes assumptions like continuous trading. Taking a position in options, therefore, provides a volatility exposure (Vega risk) that is contaminated with the direction of the underlying stock or index level.

Variance swaps emerged as a means of obtaining a purer volatility exposure. These instruments took off as a product in the aftermath of the Long Term Capital Management (LTCM) meltdown in late 1998 during the Asian crises. Some additional features are

- variance is the square of volatility;
- simple payoffs; and
- simple replication via a portfolio of vanilla options.

These are described in the next few sections.

3. The Variance Swap Mechanism

Variance swaps or variance futures (also called variance contracts) are equity derivative instruments offering pure exposure to daily realised future variance. Variance is the square of volatility (usually denoted by the Greek symbol σ^2). At expiration, the swap buyer receives a payoff equal to the difference between the annualised variance of logarithmic stock returns and the swap rate fixed at the outset of the contract. The swap rate (or delivery price) can be seen as the fixed leg of the swap and is chosen such that the contract has zero present value.

In short, a variance swap is not really a swap at all but a forward contract on realised annualised variance. A long position's payoff at expiration is equal to [DD 99]

$$VNA \left[\sigma_R^2 - K_{\text{var}} \right] \quad (1)$$

where VNA is the Variance Notional Amount, σ_R^2 is the annualised non-centered realised variance of the daily logarithmic returns on the index level and K_{var} is the delivery price. Note that VNA is the notional amount of the swap in Rand per annualised variance point. The holder of a variance swap at expiration receives VNA Rands for every point by which the stock's realised variance has exceeded the variance delivery price.

A capped variance swap is one where the realised variance is capped at a predefined level. From Eq. (1) we then have

$$VNA \left[\min(\text{cap}, \sigma_R^2) - K_{var} \right].$$

The realised variance is defined by

$$\sigma_R^2 = \frac{252}{n} \sum_{i=1}^n \left[\ln \left(\frac{S_i}{S_{i-1}} \right)^2 \right] \quad (2)$$

with S_i the index level and n the number of data points used to calculate the variance [BS 05].

If we scrutinised Eq. (2), we see that the mean logarithmic return is dropped if we compare this equation with the mathematically correct equation for variance. Why? Firstly, its impact on the realised variance is negligible. Secondly, this omission has the benefit of making the payoff perfectly additive³. Another reason for ditching the mean return is that it makes the estimation of variance closer to what would affect a trader's profit and loss. The fourth reason is that zero-mean volatilities/variances are better at forecasting future volatilities. Lastly *Figlewski* argues that, since volatility is measured in terms of deviations from the mean return, an inaccurate estimate of the mean will reduce the accuracy of the volatility calculation [Fi 94]. This is especially true for short time series like 1 to 3 months (which are the time frames used by most traders to estimate volatilities).

Note, historical volatility is usually taken as the standard deviation whilst above we talk about the variance. The question is: why is standard deviation rather than variance often a more useful measure of variability? While the variance (which is the square of the standard deviation) is mathematically the "more natural" measure of deviation, many people have a better "gut" feel for the standard deviation because it has the same dimensional units as the measurements being analyzed. Variance is interesting to scientists, because it has useful mathematical properties (not offered by standard deviation), such as perfect additivity (crucial in variance swap instrument development). However, volatility is directly proportional to variance.

³ Suppose we have the following return series: 1%, 1%, -1%, -1%. Using variance with the sample mean, the variance over the first two observations is 0, and the variance over the last two observations is zero. However the total variance with sample mean over all 4 periods is 0.01333%, which is clearly not zero. If we assumed the sample mean to be zero, we get perfect variance additivity.

4. Variance Swap Pricing in Theory

The price of the variance swap per variance notional at the start of the contract is the delivery variance K_{var} . So how do we find the delivery price such that the swap is immune to the underlying index level? Carr and Madan came up with a static hedge by considering the following ingenious argument [CM 98]: We know that the sensitivity of an option to volatility, Vega, is centered (like the Gaussian bell curve) around the strike price and will thus change daily according to changes in the stock or index level. We also know that the higher the strike, the larger the Vega. If we can create a portfolio of options with a constant Vega, we will be immune to changes in the stock or index level – a static hedge. Further, by induction, it turns out that the Vega is constant for a portfolio of options inversely weighted by the square of their strikes. This hedge is independent of the stock level and time.

From the previous argument we deduce that the fair price of a variance swap⁴ is the value of a static option portfolio, including long positions in out-the-money (OTM) options, for all strikes from 0 to infinity. The weight of every option in this portfolio is the inverse square of its strike. Demeterfi and Derman et. al. discretised Carr and Madan's solution and set out all the relevant formulas and this will be the subject of the technical note [DD 99].

5. Pricing in Practice

Here are some practical hints to consider when estimating the implied variance, K_{var} , through the portfolio of options:

1. The chosen strikes have fix increments that are not infinitesimally small. This introduces a discretisation error⁵ in the approximation of the fair variance. The smaller the increments the smaller the error. Safex will use an increment of 10 index points in the valuation of variance futures on indices.
2. A strike range is chosen and is thus also finite. This introduces a truncation error. This range is heavily dependent on the range of volatilities available. Note that the fair variance accuracy is more sensitive to the strike range, than the strike increments. In other words, the accuracy of the fair variance is more prone to truncation errors. Safex publishes volatility skews for a strike range of 70% to of 130%. The strike range used will thus be from 70% moneyness to 130% moneyness. This will ensure that Vega is constant over this range of strikes. We must be aware that even though truncation is the main error, fixing Vega to the strike range combined with optimal strike spacing, ensures that Vega is constant over that strike range.
3. For a small number of symmetrical options (< 20) we have that the lower the put bound strike the more over-estimated the fair variance. This over-estimation is less pronounced the higher the number of symmetrical options.

⁴ The fair price of a variance swap, given that it is the price of variance in the future, it is also referred to as the fair value of future variance.

⁵ We implicitly use the term error, here and not uncertainty. This is so because an error implies that the theoretical true value is known, whereas uncertainty does not.

- For a low put bound strike (< 40%) we have that the higher the number of symmetrical options, the more underestimated the fair variance. This underestimation is considerably less pronounced the higher the left wing bound.

Finding the optimal strike increments (discretisation) and the strike range (minimise truncation errors) are a trade-off between over and under estimating the fair variance.

6. Mark to Market

Any time, t , during the life of a variance swap, expiring at time T , the price of the variance swap, can be decomposed into a realised variance part (the already crystallised variance), and an implied variance part (the variance part that must still be crystallised or future/fair variance). Because variance is additive the *price of the variance swap of 1 variance notional* at time t (or the price of a new swap maturing at time $(T-t)$, is

$$V_{mm} = \frac{t}{T} \sigma_R^2 + \frac{T-t}{T} K_{var}(t, T) \quad (3)$$

At contract initiation the value will be derived from 100% implied variance, as the contract moves closer to expiry realised variance will play an increasingly important role. This is shown in Figure 3 below. Variation margin is thus the difference between the value of the variance future today and what it was yesterday as given by Eq. (3).

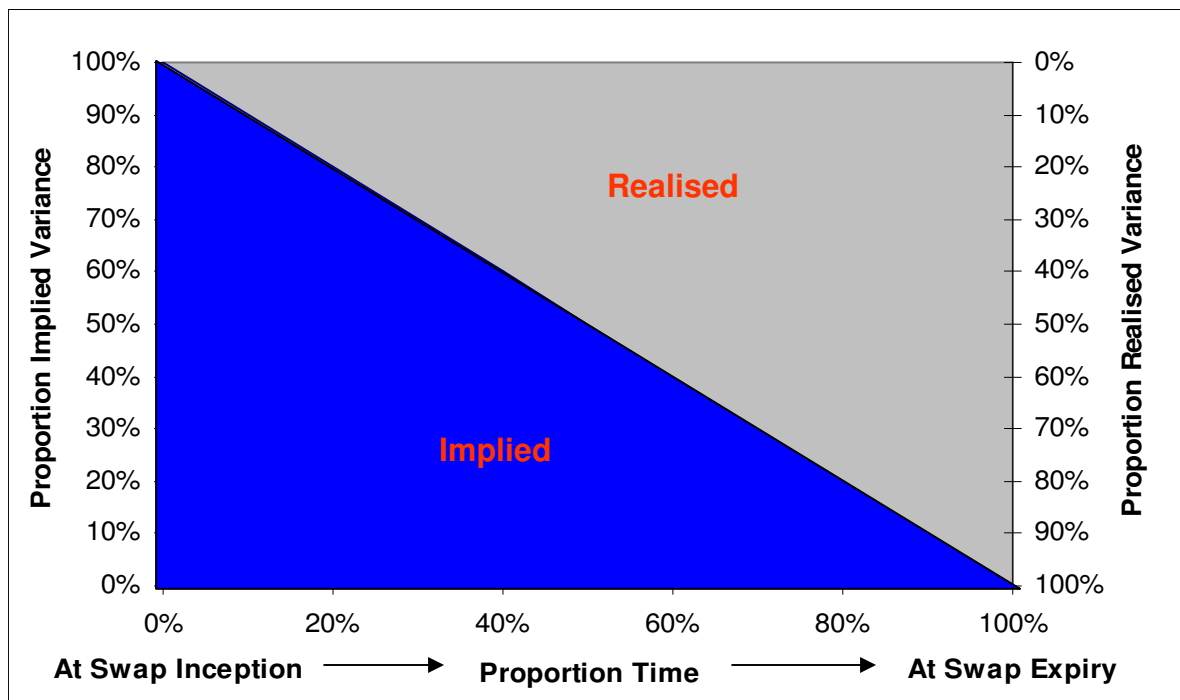


Figure 3: As time passes, realised variance start to play the dominant role in the value of a variance future.

7. Initial Margin Requirements

The initial margin requirement is determined in using the underlying's historical data to determine a volatility of volatility like parameter, the Risk Parameter λ . The Risk Parameter is currently set at 10%. The initial margin requirement (IMR) per contract for a variance future is then given by

$$IMR = \lambda * K_{market} * VPV . \quad (4)$$

Here, K_{market} is the market value of implied variance and VPV is the Variance Point Value. Note, the VPV is a quantity similar to the current R10 per point used for index futures. It is currently set at $VPV = R1$ for the Safex variance futures.

Most traders trading a variance future usually start with a Vega Rand amount they want to hedge – this might be due to a book of options. We can then define [Wi 08]

$$VNA = \frac{VA}{2\sqrt{K_{var}}} = C * VPV \quad (5)$$

with VA the Vega amount to be hedged in Rand. Here, K_{var} is in absolute terms e.g., if the volatility is 30% then $K_{var} = 30^2 = 900$. From this

$$C = \frac{VNA}{VPV} = \frac{VA}{2\sqrt{K_{var}} VPV} . \quad (6)$$

Eq. (6) is used to determine the number of contracts, C , to trade.

From Eq. (3) and Figure 3 we deduce that the risk in a variance future dwindles as we approach the expiry date – there is more certainty in the outcome because of the realized variance part. For 3 months and 6 months variance future the initial margins are then reduced in accordance with the tables in Table 1 and Table 2 respectively.

		Margin as % of IMR
Month	0 to 1	100%
Month	1 to 2	75%
Month	2 to 3	50%

Table 1: Reduction in initial margin requirements for a 3 month variance future.

		Margin as % of IMR
Month	0 to 1	100%
Month	1 to 2	90%
Month	2 to 3	80%
Month	3 to 4	70%
Month	4 to 5	60%
Month	5 to 6	50%

Table 2: Reduction in initial margin requirements for a 6 month variance future.

8. Profit and Loss

To determine the profit and loss for a variance future, we return to Eq. (1) and (3). We then have

$$PL = VNA[V_{mtm} - K] = C * VPV [V_{mtm} - K] \quad (7)$$

where V_{mtm} is the daily mark-to-market variance level as calculated using Eq. (3) and set by Safex on a daily basis. K is the delivery price or strike price set at initiation of the contract.

9. Trading a Standard Variance Future

Eq. (6) determines the number of contracts to trade if a certain Rand amount of Vega exposure needs to be hedged. This holds at the initiation of the variance futures contract. Also, remember that at initiation the value of the variance future is zero. The delivery price, is then found from Eq. (1).

An investor buying into an existing variance future (after initiation) will buy an instrument that already has value due to the realised variance part in Eq. (2). His future's value needs to be zero as well but the delivery price is fixed. The only way to get a value of zero is to change the number of contracts to trade. The number of contracts is then given by

$$C_{new} = \frac{VA}{2 * VPV * \frac{V_{mtm}}{\sqrt{K}}} \quad (8)$$

where V_{mtm} is the daily mark-to-market variance level as calculated through Eq. (3) and set by Safex on a daily basis. K is the delivery price or strike price set when the existing variance future was initiated. VA is the Vega Rand amount to be hedged.

10. References

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Annexure A: Example of a Variance Future Term Sheet

JSE EQUITY DERIVATIVES MARKET NOTICE

Number

Date **24 July 2008**

New Variance Futures

The following new future has been added to the list with immediate effect and will be available for trading on Friday 25 July 2008:

Summary Contract Specifications

Futures Contract	
Code	XXXX
Underlying Instrument	FTSE/JSE TOP40 Index future
Variance Point Value, VPV	1 Variance point = 1 Rand
Trade Date	14 August 2008
Expiry Dates & Times	17:00 on 18 September 2008
Quotations	Variance points to two decimals.
Minimum Price Movement	R1 (0.01 in variance points)
Expiry Valuation Method	Realised variance (RV) as calculated by the JSE Limited over the contract period (Formula Appendix A)
Final Equity Payment on Expiry	$C * VPV * [RV - K]$
Clearing House Fees	TBA
Initial Margin per Contract	R90
Class Spread Margin	R
V.S.R	3.5
Volatility Strike, VK	30
Volatility Cap, VC	NA

Variance Strike, K (K = VK²)	900
Variance Cap, KC	NA
Number of trading observation days	All trading days between and inclusive of the trade and expiration dates
Communication Code	

Appendix

Realised variance will be calculated as follows:

$$RV = 10000 \times \frac{252}{n-1} \sum_{i=1}^{n-1} \left[\ln \left(\frac{S_{i+1}}{S_i} \right)^2 \right]$$

Where:

i = each observation date

n = number of observation dates as of trade date

S_i = the closing level of the underlying index
future on the i -th observation date

S_n = the closing level on the exchange of
the index future on the expiration date