Volatility Skew FAQs

Antonie Kotzé[†], Angelo Joseph[†], Nolene Naido[‡], James Boardman[‡] and Magnus de Wet[‡]

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What is a volatility skew?

The Black and Scholes model assumes that volatility is constant. This is at odds with what happens in the market where traders know that the formula misprices deep in-the-money and deep out-the-money options. The mispricing is rectified when options (on the same underlying with the same expiry date) with different strike prices trade at different volatilities - traders say volatilities are skewed when options of a given asset trade at increasing or decreasing levels of implied volatility as you move through the strikes.

The empirical relation between implied volatilities and exercise prices is known as the "volatility skew". The volatility skew can be represented graphically in 2 dimensions (strike versus volatility). The volatility skew illustrates that implied volatility is higher as put options go deeper in the money. This leads to the formation of a curve sloping downward to the right. Sometimes, out-the-money call options also trade at higher volatilities than their at-the-money counterparts. The empirical relation then has the shape of a smile, hence the term "volatility smile". This happens most often in the currency markets.

Why is there a volatility skew?

In 1972 Black and Scholes mentioned in a paper "the historical estimates of the variance include an attenuation bias – the spread of the estimated variance is larger than the true variance". This would imply that for securities with a relatively high variance (read volatility), the market prices would imply an underestimate in the variance, while using historical price series would overestimate the variance and the resulting model option price would be too high; the converse is true for relative low variance securities. Black and Scholes further showed that the model performed very well, empirically, if they use the right variance. In 1979, Macbeth and Mervile

[†] Consultants from Financial Chaos Theory – see http://www.quantonline.co.za

[‡] From Safex – see http://www.safex.co.za/ed/

extended this empirical research of Black and Scholes and also showed that the skew existed. At that point in time the skew wasn't pronounced but the market crash of October 1987 changed all of that.

If one looks at option prices before and after October 1987, one will see a distinct break. Option prices began to reflect an "option risk premium" - a crash premium that comes from the experiences traders had in October 1987. After the crash the demand for protection rose and that lifted the prices for puts; especially out-the-money puts. To afford protection, investors would sell out-the-money calls. There is thus an over supply of right hand sided calls and demand for left hand sided puts - alas the skew.

The real phenomenon underlying volatility skews is that either, market imperfections systematically prevent prices from taking their true Black & Scholes values, or the underlying asset price process differs from the lognormal diffusion process assumed by the Black and Scholes model. The volatility skew is thus the market's way of getting around Black and Scholes's simplifying assumptions about how the market behaves.

What is the at-the-money term structure of volatility?

Another aspect of volatility that is observed in the market is that at-the-money options with different expiries trade at different volatilities. The at-the-money volatilities for different expiry dates are decreasing in time. This provides another method for traders to gauge cheap or expensive options.

The term structure of volatility arises partly because implied volatility in short options changes much faster than for longer options and partly due to the assumed mean reversion of volatility. The effect of changes in volatility on the option price is also less the shorter the option.

What is a volatility surface?

Combining the ATM term structure of volatility and the skew per expiry date, will render a 3 dimensional graph (time to expiry versus strike versus volatility). This is known as the volatility surface.

What is relative volatility and how is it calculated?

The relative volatility is the difference in volatility from the ATM volatility and is obtained by taking "volatility of strike – ATM volatility". The relative volatility of the ATM strike is 0%. For example if the ALSI ATM volatility moves from, say 30% to 29% we calculate the new volatility for each strike with the following formula: New volatility = New ATM + Relative Volatility of that strike. For a strike with moneyness of 95% and relative volatility of 2%, the New Volatility = 29% + 2% = 31%. There is an example of this in the Excel spreadsheet at the bottom of this page.

What does sticky strike mean?

In a sticky strike model ("absolute skew"), the implied volatility of each option is constant as the spot changes. Another way to put this is that skew is kept fixed at strikes as the spot is shifted. This means the volatility is independent of the spot, it depends on the strike only. A sticky strike skew plots volatility against actual strikes. Intuitively, “sticky strike” is a poor man’s attempt to preserve the Black-Scholes model. It allows each option an independent existence, and doesn’t worry about whether the collective options market view of the spot is consistent.

What does sticky delta mean?

In the sticky delta model (also known as a relative or floating skew), the implied volatility depends on the moneyness only (spot divided by strike). The ATM implied volatility does not change as the underlying spot changes. This entails that the smile floats with moneyness as spot is shifted such that the delta of options are preserved. In this model, moneyness is plotted against relative volatility (difference in volatility from the ATM volatility). The sticky-delta rule quantifies the intuition that the current level of at-the-money volatility – the volatility of the most liquid option – should remain unchanged as spot changes.

What type of volatility skew is used for the ALSI?

Some market participants use a sticky strike, and some a sticky delta, surface. To obtain an average skew, all skews are converted to sticky delta skews. The average sticky delta surface is then converted into a sticky strike surface. This is fed into Nutron, which returns a sticky delta skew. The sticky delta skews float with changes in the ATM volatilities.

Which is better: sticky strike or sticky delta?

There is no conclusion yet. Rubinstein and Jackwerth in 1997 compared several models and found that sticky-strike best predicts future smiles. However, Derman found in 1999 that market conditions should set the tone. He found that if the markets are trending, where the market is undergoing significant changes in levels without big changes in realized volatility, sticky delta rules. However, sticky strike rules if the market trades in a range.

Daglish, Hull and Suo concluded in 2006 that all versions of the sticky strike rule are inconsistent with any type of volatility smile or volatility skew. They state that “If a trader prices options using different implied volatilities and the volatilities are independent of the asset price, there must be arbitrage opportunities.” They further found that the relative sticky delta rule can be at least approximately consistent with the no-arbitrage condition.

How is the ALSI volatility skew shifted?

On the first Monday of a new month, we send out an email to the options market, asking for their volatility skews. The result is an average of the 9 skews we receive. All skews are converted into sticky delta skews. The average surface is obtained. This is converted into a sticky strike surface having 9 strikes ranging from 70% to 130% in moneyness - the middle strike is at the money, with moneyness 100%.

Once this is fed into Nutron, it is converted into a *sticky delta format*, with moneyness plotted against relative volatility. The relative volatility = volatility of that strike – ATM volatility. The relative volatility of the ATM strike is 0%. For example if the ALSI ATM moves from, say 30% to

29% we calculated the new volatility for each strike with the following formula: New volatility = New ATM + Relative Volatility of that strike. For a strike with moneyness of 95% and relative volatility of 2%, the New Volatility = 29% + 2% = 31%. Thus the skew will only move up or down to accommodate the change in the ATM volatility. A worked example to help illustrate the process can be found at the bottom of this page.

How often is the sticky delta skew shifted?

On a daily basis, as the ATM volatility changes.

How often is the sticky strike volatility surface updated?

Monthly. This process will be more frequent later this year.

How is interpolation done?

Linear. The skew actually looks like straight lines joined together, when you zoom in. Outside of the 9 points, we extrapolate with a minimum volatility of 5% and a max of 65%. The minimums and maximums are published with our volatility skews. There is a link to our volatility surface below.

How does the JSE get the ATM volatility?

The ATM volatility is read off the sticky strike skew.

How is volatility rounded?

All volatilities are rounded to 2 decimal places.

Where can I find daily ATM volatilities?

You can find the ATM Volatilities on a daily basis here:

http://www.safex.co.za/PUB/mtmdata/

Where can I find the latest volatility skew?

The URL for this month's (July 2009) volatility skew:

http://www.safex.co.za/PUB/mtmdata/Vol%20Skew%20Indices/Vol%20Skew%20Changes%20Jul09.xls

Are older ones kept?

You can find all our previous volatility skews here:

http://www.safex.co.za/PUB/mtmdata/Vol%20Skew%20Indices/

ALSI Volatility Skew Example

Relative volatility = Volatility of strike - ATM Volatility

Strikes	Volatility	Moneyness	Relative volatility
25000	40%	84%	10%
24000	38%	88%	8%
23000	36%	91%	6%
22000	32%	95%	2%
21000	30%	100%	0%
20000	27%	105%	-3%
19000	25%	111%	-5%
18000	22%	117%	-8%
17000	20%	124%	-10%

If ATM volatility moves from 30% to 29%

New volatility = New ATM + Relative volatility

Strikes	New Volatility
25000	39%
24000	37%
23000	35%
22000	31%
21000	29%
20000	26%
19000	24%
18000	21%
17000	19%

Net result: graph has moved down by 1%