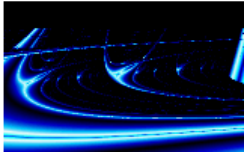


CONSTRUCTING A SA INDEX VOLATILITY SURFACE from Exchange Traded Data

Dr A. A. Kotzé
Financial Chaos Theory
February 2011

Saggitarius A*: supermassive black
hole at the Milky Way's center





Niels Bohr
and
Albert Einstein



Before I came here I was confused about the subject. Having listened to your lecture I am still confused. But on a higher level.

Enrico Fermi (1901-1954)

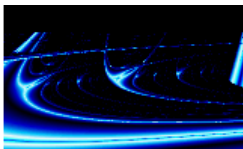
The Happy Face of Volatility



Myron Scholes (1941 -)



Robert Merton (1944 -)

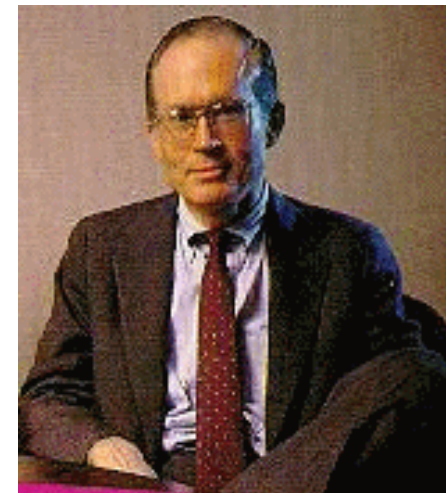


Whenever I see your smiling face
I have to smile myself
Because I love you (Yes, I do)
And when you give me that pretty little pout
It turns me inside out
There's something about you, baby (I don't know)

James Taylor – “Your Smiling Face” © Country Road Music

The smile on your face
Lets me know that you need me
There's a truth in your eyes
Saying you'll never leave me
The touch of your hand
Says you'll catch me if ever I fall
You say it best
When you say nothing at all

Keith Whitley, Allison Kraus – “When You Say Nothing at All” © RCA

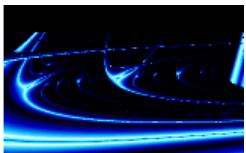
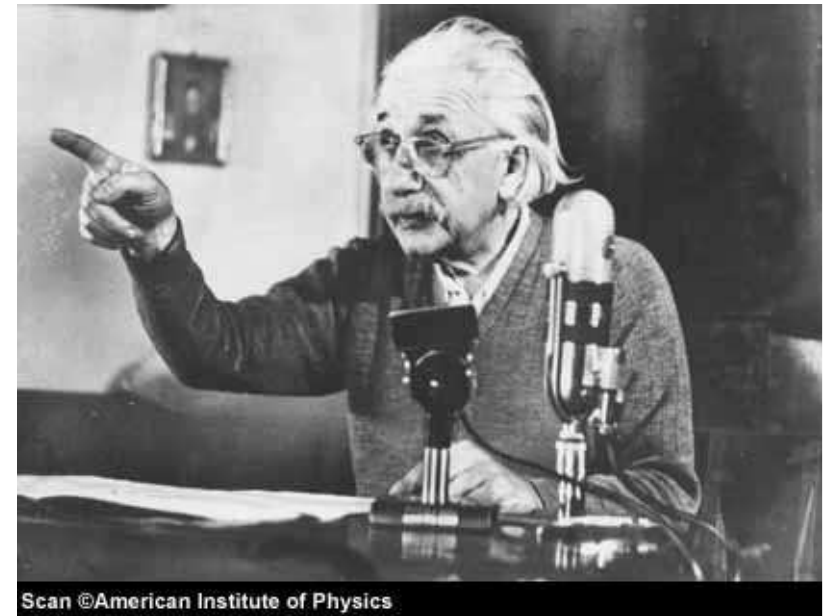


Fischer Black (1938 - 1995)

FINANCIAL
CHAOS
THEORY

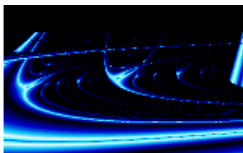
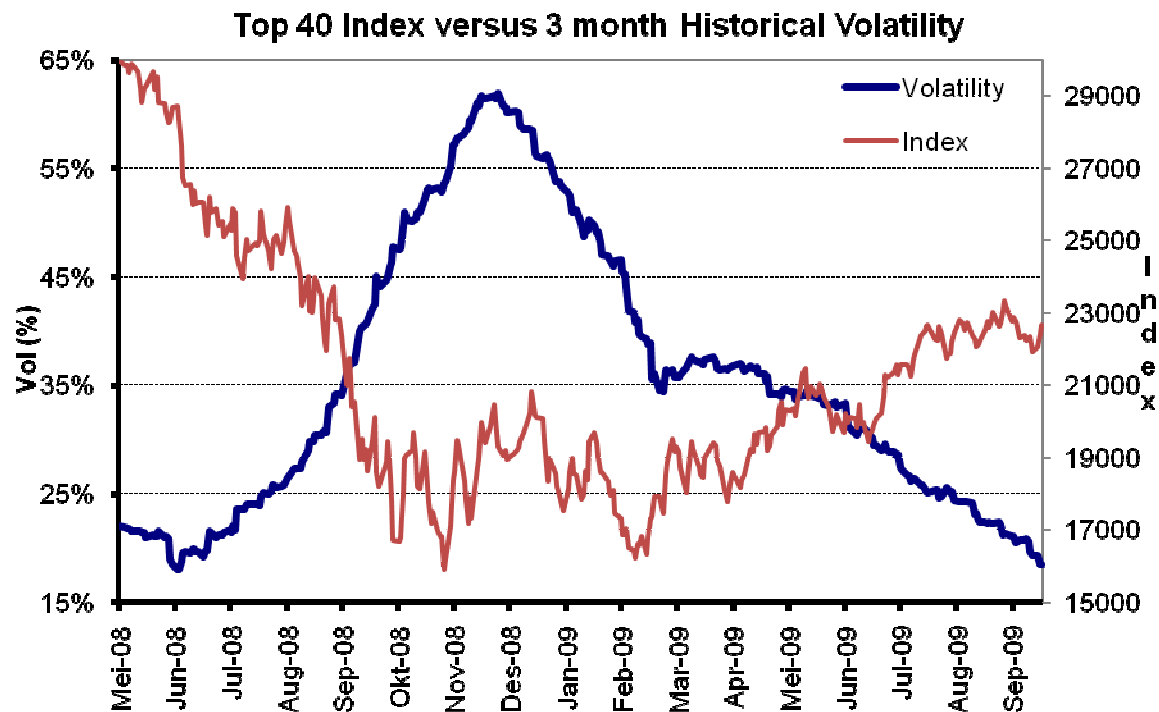
“Happy” smiles

- These song writers obviously knows nothing about volatility smiles.
- Options “smile” at traders, but that smile is not a source of happiness
- Will the smile catch you if you fall? Probably not.



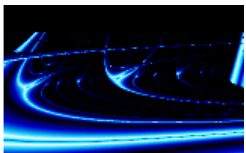
Volatility Dynamics?

- The Black & Scholes option pricing model assumes that volatility is constant.
- It is well known, that when equity prices go up (down) volatility usually goes down (up) – there is an inverse relationship between volatility and the underlying asset price.



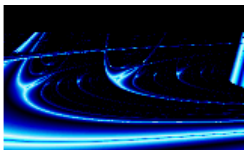
The Volatility Skew

- The Black and Scholes model assumes that volatility is constant.
- However, traders know that the formula misprices deep in-the-money and deep out-the-money options.
- The mispricing is rectified when options (on the same underlying with the same expiry date) with different strike prices trade at **different volatilities**
- **We say volatilities are skewed when options of a given asset trade at increasing or decreasing levels of implied volatility as you move through the strikes.**
- The empirical relation between implied volatilities and exercise prices is known as the “volatility skew/smile”.
- The volatility skew can be represented graphically in 2 dimensions (strike versus volatility).



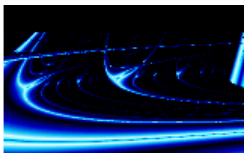
Observing the Smile/Skew

- The volatility skew (smile) was first observed and mentioned by Black and Scholes in a paper that appeared in 1972.
- It was then empirically described in 1979 by Macbeth and Merville.
- At that point in time it wasn't pronounced but the market crash of October 1987 changed all of that.



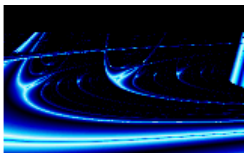
The Crash of 1987

- If one studies option prices before and after October 1987, one will see a distinct break.
- Option prices begin to reflect an "option risk premium" --- a crash premium that comes from the experiences traders had in October 1987.
- After the crash the demand for protection rose and that lifted the prices for puts; especially out-the-money puts. To afford protection, investors would sell out-the-money calls.
- There is thus an over supply of right hand sided calls and demand for left hand sided puts --- alas the skew. In fact, out-the-money put options usually trade at a premium to out-the-money calls.



The Reality

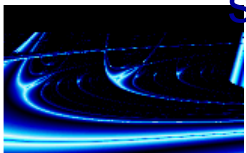
- The skew is a reality and every option trader needs to get to grips with it.
- The smile phenomenon has spread to commodity options, interest-rate options, currency options and almost every other volatility market.
- Since the Black-Scholes model cannot account for the skew, option traders and risk managers have begun to use more complex models to value and manage options.
- Some of these methods include generating a market related implied volatility surface that accurately reflects the correct options values.



Shapes of the Skew

3 Distinct Shapes

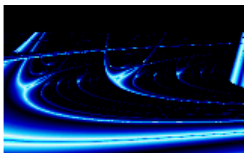
- **Supply Skew**: The supply skew is defined by higher implied volatility for lower strikes and lower volatility for higher strikes. Supply refers to the natural hedging activity for the major players in the market who have a supply of something they need to hedge. Stock Index and interest rate markets have supply skews. The natural hedge for stockowners is to buy puts in order to protect the value of their "supply" of stock and sell calls to offset the puts' prices - collars. This natural action in the marketplace determines the structure of the skew.
- **Demand Skew**: Demand skews have higher implied volatility at higher strikes and lower implied volatility at lower strikes. The natural hedger in demand markets is the end user. The "collar" for a demand hedger is to buy calls and sell puts. The grain markets and energy markets are good examples of demand skews.



Shapes of the Skew

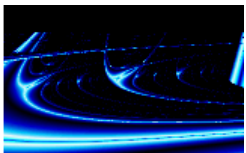
- **Smile Skew**: The third type of skew is called a smile skew. The smile skew is illustrated by higher implied volatility as you move away from the at the money strike. The at the money strike would have the lowest implied volatility while the strikes moving up and down would have progressively higher implied volatility.

The smile skew is generally only observed in the currency markets. The natural hedger has to hedge currency moves in either direction depending of whether they have accounts payable or receivable in the foreign currency.



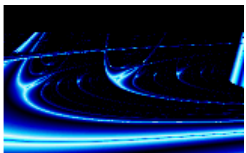
Exchanges and Market Makers

- In liquid markets, exchanges value all options off their MtM levels
- Exchanges like CME employ market makers who will supply a MtM level for every option with open interest
- Safex has a different model. There are no market makers
- Every option with open interest has to be valued at the end of every day for margin purposes; even if that option did not trade during the day
- How does Safex do that?
- Safex can use the last traded price
- Sometimes, options trade only once
- Risk management becomes an issue!



Safex: the early days

- The exchange traded option market started in 1992
- All options valued off a flat implied volatility
- Implied Volatility found by looking at last traded option (per expiry), nearest the money (less than 100 points from spot)
- ATM volatility found for every expiry of every underlying
- All options, irrespective of strike, valued using the ATM vols



Safex: Introduction of Skew

105 CENTRAL STREET, HOUGHTON ESTATE 2198 PO BOX 95709 GRANT PARK 2051
TEL. (27-11)728-5960 FAX. (27-11)728-5970 INTERNET: URL: <http://www.safex.co.za>



NOTICE TO ALL MEMBERS

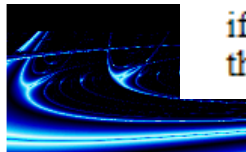
NOTICE 1046

22 November 2000

MARKING-TO-MARKET THE OPTIONS SKEW

Safex have marked-to-market using flat volatility since options were introduced in 1992. However, it is becoming increasingly apparent that this approach needs to be reviewed.

We are suggesting introducing a system whereby we fix the volatility skew on a once a week basis for all expirations of the index contracts. We will continue to obtain a daily volatility mark-to-market for the at-the-money options and then imply the skew. Therefore we would ask for quotes at (for example) 100, 200, 500 & 1000 points in/out-the-money on Monday. The shape of the skew on Monday would be maintained for the following five days and would move with the at-the-money volatility (therefore if the at-the-money options are 16% on Monday and 100 points in-the-money is 16.25%, when the at-the-money moves to 16.50%, the 100 points in-the-money will move to 16.75%).



Safex: Introduction of Skew

NOTICE TO ALL MEMBERS

NOTICE 1051

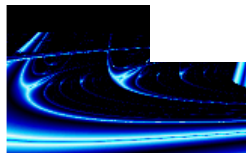
29 November 2000

VOLATILITY SKEW

Further to notice 1046, we have received considerable feedback regarding the possible introduction of marking-to-market with volatility skew. There is broad consensus that this is an extremely good move for the market. This view was endorsed at the executive committee meeting on 29th November 2000, when they approved the move to marking-to-market using the skew.

However, there are two major issues still outstanding:

1. We need to define and approve a system for how to arrive at the skew. As many people know we have had several problems arriving at consensus on the a-t-m volatility's, and the introduction of a new system must take cognisance of the previous issues. The two choices appear to be implying the skew from actual reported trades (which will require accurate reporting of volatility's on negotiated trades) or introducing some sort of regular open auction facilitated through Safex;
2. We need to consider when to introduce the change and whether it can/should be introduced for contracts with existing open interest, that may have been priced in anticipation of flat volatility on mark-to-markets. The timing of the introduction is of critical importance and consideration must be given to both the legal and practical implications of a change.



Safex: Introduction of Skew

NOTICE TO MEMBERS

NOTICE 1072

8 March 2001

IMPLEMENTATION OF THE SKEW IN MARK-TO-MARKET

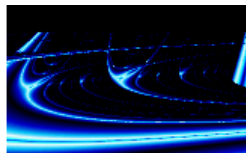
Further to a meeting of traders held yesterday, it has been decided that the introduction of the skew into the mark-to-market and margining systems will be implemented as follows:

1. The March 01 and June 01 products shall continue to operate using flat at-the-money skew for mark-to-market and margining purposes;
2. The March 02 and March 03 contracts on which there is existing open interest shall continue to be marked-to-market on flat at-the-money volatility but shall start to be margined using the skew;
3. All other products shall use the skew in both the mark-to-market and margining processes.

This new treatment of the March 02 and March 03 contracts overcomes the concerns regarding possible systemic risk, whilst ensuring that those who have existing positions in the contracts are not unduly prejudiced.

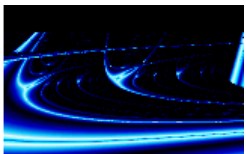
The planned roll out of the changes is as follows:

1. Specifications for the required changes to margining calculations shall be distributed within the next 7 days;
2. Testing of the resultant numbers shall be ongoing throughout March;
3. Implementation of the new system on the March 02 and 03 contracts will be at the beginning of April;
4. Implementation of the remaining skew changes shall be at the beginning of May.



Safex: June 2001

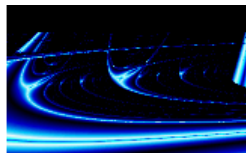
- Index volatility surface introduced on 1 June 2001
- Bond volatility surfaces introduced on 13 August 2003
- Proposal was to obtain skews through an auction process
- Notice 1095 stated “The objective of the auction is to determine a single fair price for a option strike before trading occurs. During the auction period, members may place bids and offers for the individual contract on the screen.”
- The auction process was not accepted by the market with few players giving their volatilities through.
- Safex resorted to polling the market by email on a monthly basis



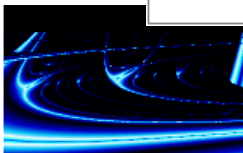
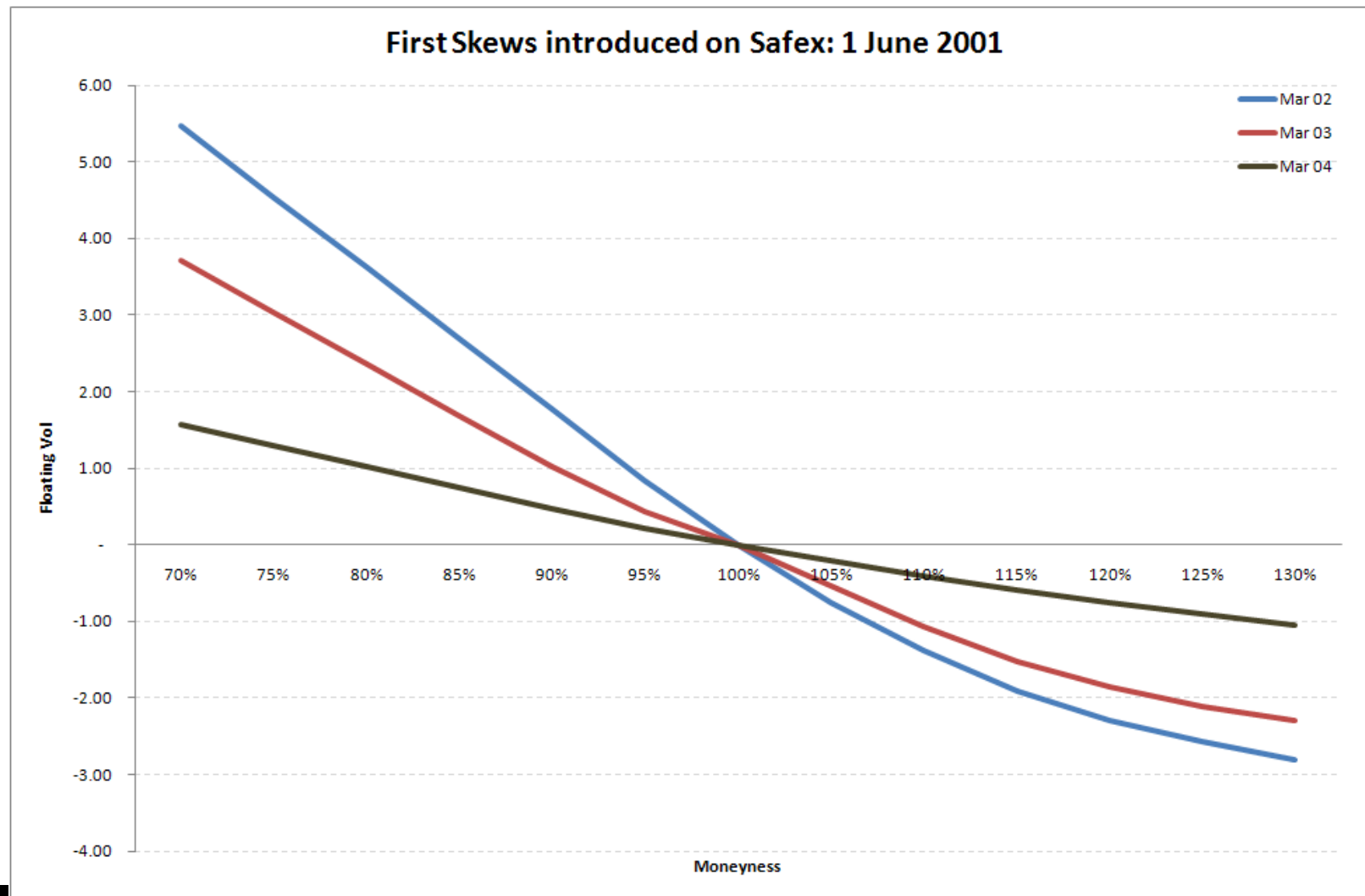
Determining the Skews: Notice F271 – 8 April 2003

The methodology of determining the skew will be as follows:-

- 1 An auction process will be initiated, wherein various participants will be polled via e-mail on a monthly basis.
2. The JSE will select applicable strikes and distribute a template for completion by the market participants.
- 3 Should there be greater than 6 respondents the highest and lowest respondents will be eliminated. An arithmetical average (rounded to 4 decimal places) of the remaining respondents will be taken.
- 4 Should there be 6 or less than 6 respondents an arithmetical average will be taken.
- 5 With the above information the JSE will determine the shape of the volatility skew.
 - 5.1 On a daily basis the at-the-money (ATM) volatility will be determined.
 - 5.2 This ATM and strike will then be utilised as the fulcrum upon which the volatility skew will be based.

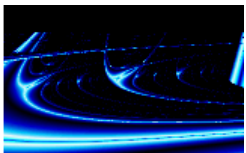


First Skews: ALSI



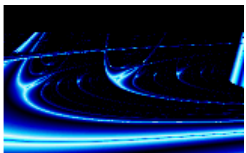
Polling the Market

- Polling the market was problematic
- Not enough participants
- Before May 2008, market complained about surface
- Polling was irregular
- Surface was stale for long periods of time

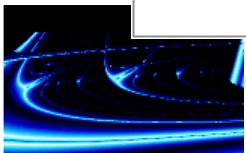
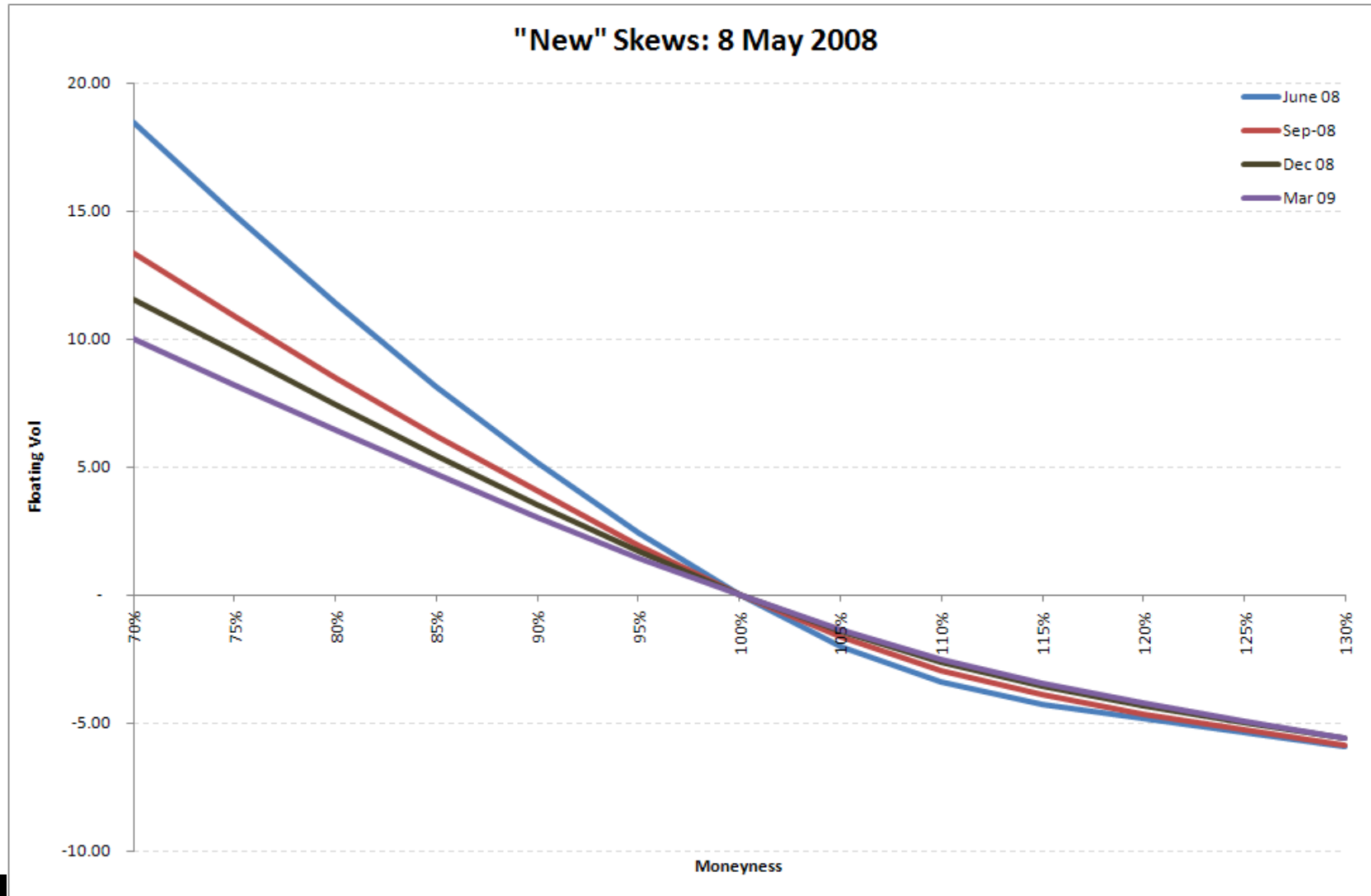


Polling the Market: renewed efforts

- Renewed effort from May 2008
- 8 Participating banks: SBSA; Deutsche; Macquarie; UBS; Investec; Absa; Nedbank; RMB-MS (later Credit Suisse but UBS vanished after 2008 crisis)
- 1 Broker: Peregrine
- Every month between 6 and 9 participants
- Used the simple average of all supplied skews – no exclusions
- Surfaced changed first Wednesday of every month
- Market accepted surface

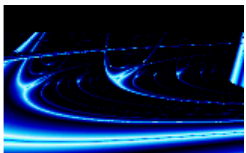


Alsi May 08 Skews



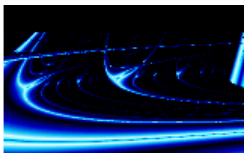
Generating a Skew

- Question: can a skew be generated from traded data?
- Safex sees all trades, why not use that?
- Before Nutron, Safex did not have all information
- Option booking methodology was problematic
- Traders only needed to supply the option premium – option match on premium. The futures level or volatility traded at was not compulsory
- During July 2009, everything changed
- Nutron forces option traders to supply 2 of the 3 quantities: premium, volatility, futures level traded at



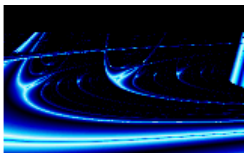
Research

- If you want to generate a skew, you need a model
- Which one: stochastic or deterministic?
- After much research, Safex decided on the deterministic route
- Reasons:
 - keep things as simple as possible - replicability
 - A road trip to London confirmed the deterministic route
 - Jim Gatheral's book and course notes (AIMS Feb 2009)
 - Two seminal research papers:
 - Dumas, Fleming and Whaley – Implied volatility Functions: Empirical Tests
 - Tompkins – Implied Volatility Surfaces: Uncovering Regularities for Options on Financial Futures



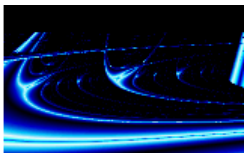
Benefits of Deterministic Models

- No assumptions about the dynamics of the underlying asset that generated the volatility
- They allow one to model volatility separately in expiry time and strike. This means that each expiry's skew is independently calibrated minimising compounding errors across expiries. This property is useful in modelling volatility surfaces in illiquid markets where data is sparse across strikes.
- Pricing using deterministic volatility preserves the **no-calendar spread arbitrage** market because no assumptions about the underlying process is made.
- The whole surface can be calibrated with minimal model error.





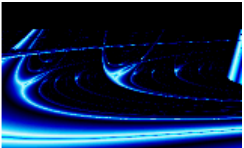


Tompkins Research

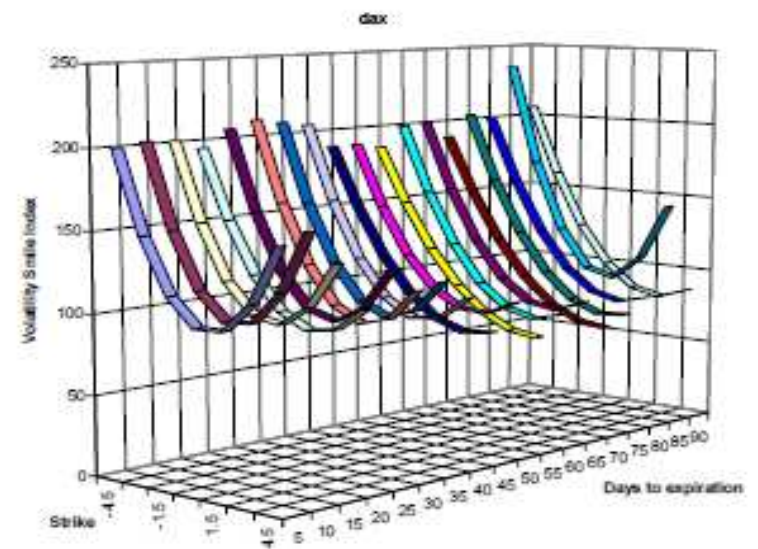
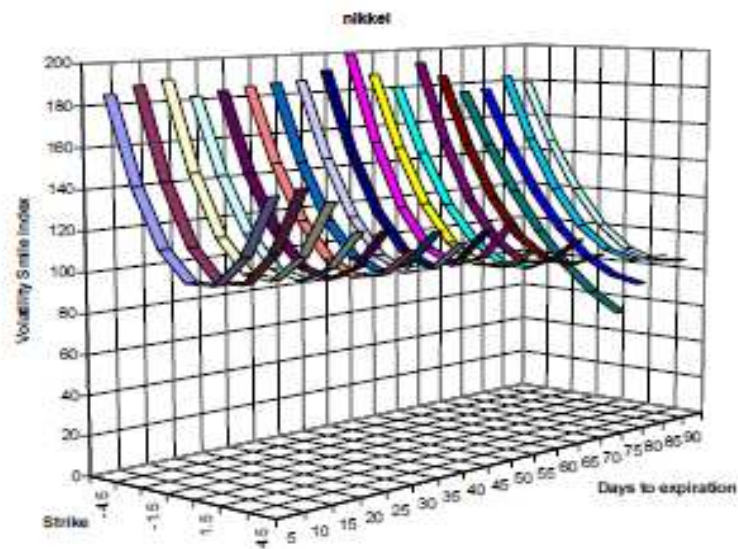
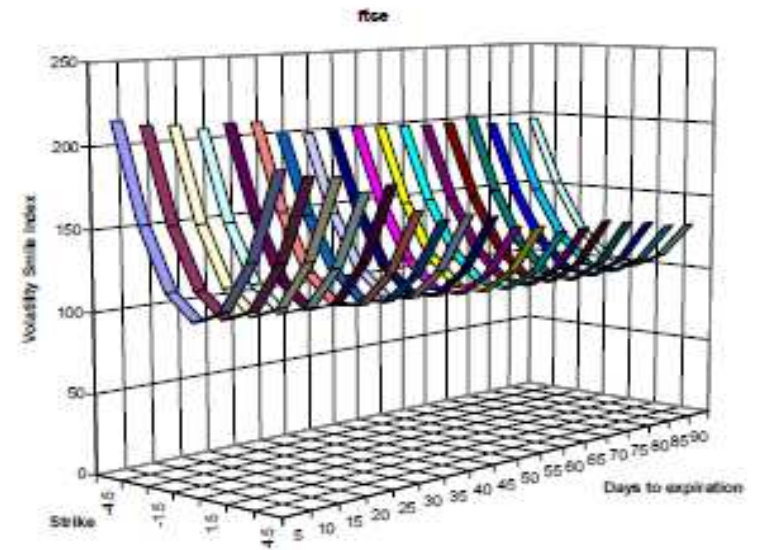
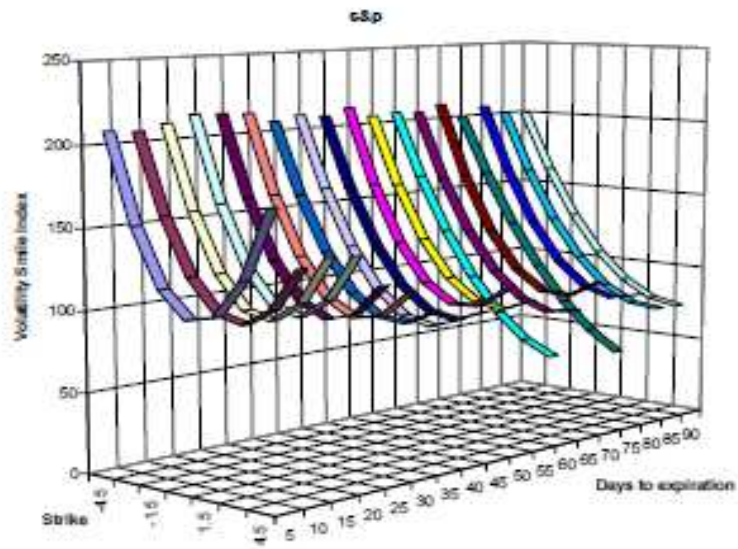
- Studies the implied volatility surfaces across different markets
- Motivation: assign economic significance to the functional form of the smile patterns
- Claim to have “all publically available data” between dates as shown
- Data obtained from relevant exchanges
- Number of option prices examined across the 16 markets was 1,862,473
- Most comprehensive empirical study to date!



The Tompkins Study

	<u>Underlying Asset</u>	<u>Time Period of Analysis</u>
Equities 	S&P 500 Futures	25/03/1986 - 24/12/1996
	FTSE Futures	02/01/1985 - 20/12/1996
	Nikkei Dow Futures	25/09/1990 - 16/12/1996
	DAX Futures	02/01/1992 - 20/12/1996
Bonds 	Bund Futures	20/04/1989 - 21/11/1996
	BTP Futures	11/10/1991 - 21/11/1996
	Gilt Futures	13/03/1986 - 22/11/1996
	US T-Bond Futures	02/01/1985 - 15/11/1996
Currencies 	Deutsche Mark / US Dollar	03/01/1985 - 09/12/1996
	British Pound / US Dollar	25/02/1985 - 09/12/1996
	Japanese Yen / US Dollar	05/03/1986 - 09/12/1996
	Swiss Franc / US Dollar	25/02/1985 - 09/12/1996
FRAs  	Euro Dollar	27/06/1985 - 16/12/1996
	Euro Sterling	05/11/1987 - 18/12/1996
	Euro D-mark	11/03/1990 - 16/12/1996
	Euro Swiss Franc	15/10/1992 - 16/12/1996

Tompkins Results for Equity Derivatives



The Tompkins Functional Form

- **A QUADRATIC EQUATION**

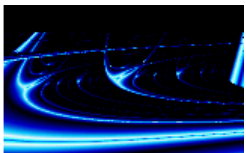
From BS
equation

$$VSI = \alpha + \beta_1 \cdot \frac{\ln(X_\tau / F_\tau)}{\sigma \sqrt{\tau / 365}} + \beta_2 \cdot \left[\frac{\ln(X_\tau / F_\tau)}{\sigma \sqrt{\tau / 365}} \right]^2 + \varepsilon$$

- **In school terms: a parabola**

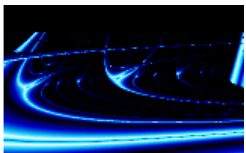
"If the sole objective was to fit a curved line, this has been achieved".

Tompkins: 2001



Gatheral's Influence

- Gatheral facilitated a short course on volatility skews at AIMS during February 2009
- We had extensive discussions with him
- We followed the following route:
 - Each expiry will be fitted with a quadratic equation
 - Skews are linked across time by another functional form: term structure
 - Leaned on Beber who studied the MIB30 (and others who studied Sweden, India and other markets)



The Safex Functional Form for each Expiry

$$\sigma_{model}(\beta_0, \beta_1, \beta_2) = \beta_0 + \beta_1 K + \beta_2 K^2. \quad (3)$$

- K is the strike price in moneyness format (Strike/Spot),
- β_0 is the constant volatility (shift or trend) parameter, $\beta_0 > 0$. Note that

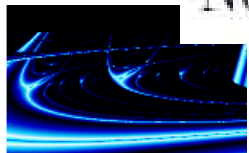
$$\sigma \xrightarrow{K \rightarrow 0} \beta_0,$$

- β_1 is the correlation (slope) term. This parameter accounts for the negative correlation between the underlying index and volatility. The no-spread-arbitrage condition requires that $-1 < \beta_1 < 0$ and,
- β_2 is the volatility of volatility ('vol of vol' or curvature/convexity) parameter. The no-calendar-spread arbitrage convexity condition requires that $\beta_2 > 0$.

Arbitrage.....

Arbitrage.....

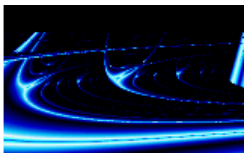
Note that Eq. (3) is also linear in the wings as $K \rightarrow \pm\infty$.



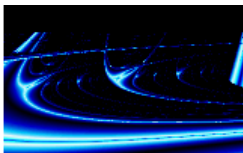
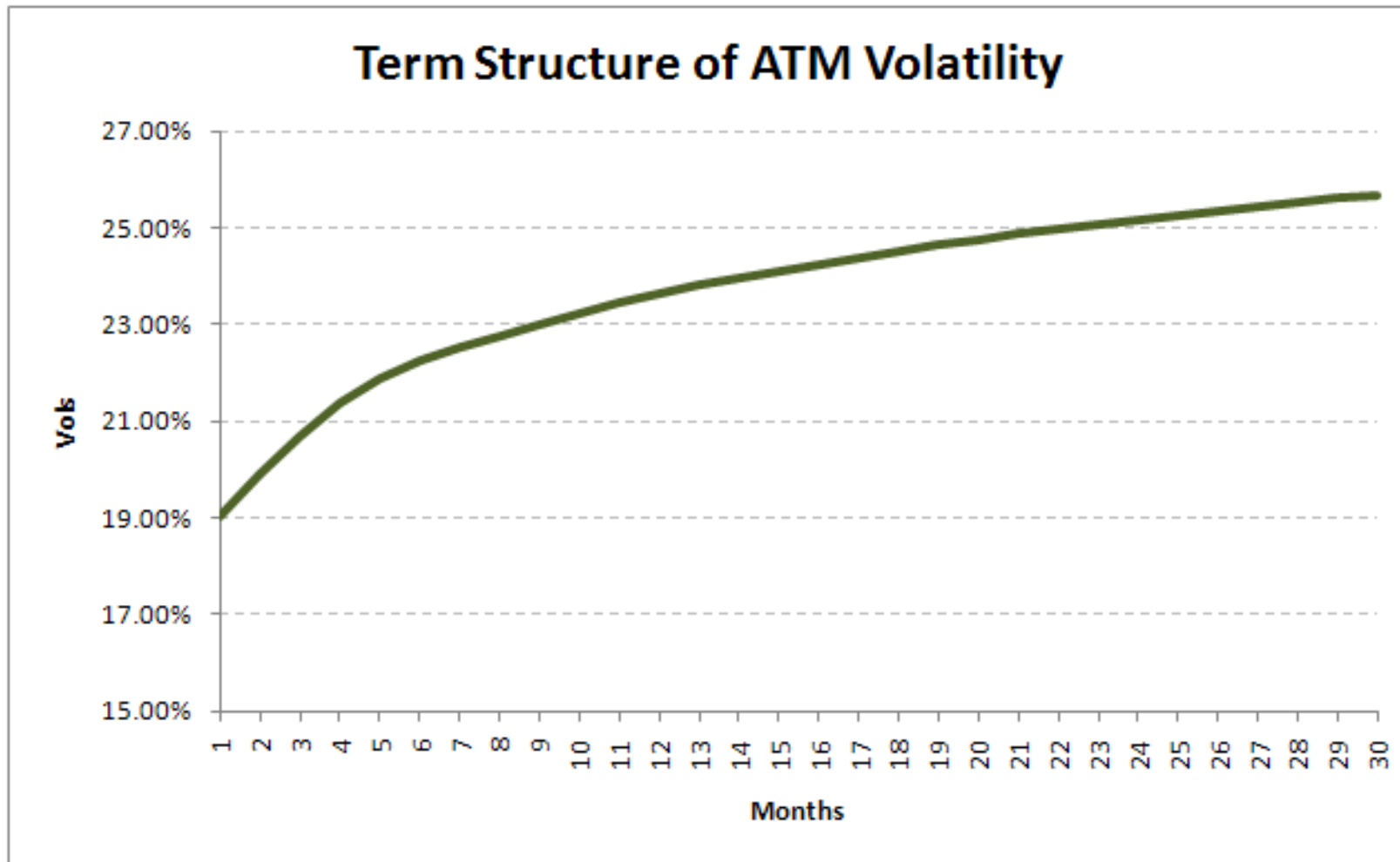
Linking across Time: the volatility term structure

$$\sigma_{atm}(\tau) = \frac{\theta}{\tau^\lambda}. \quad (4)$$

- τ is the months to expiry,
- λ controls the overall slope of the ATM term structure; $\lambda > 0$ implies a downward sloping ATM volatility term structure (this is plotted in Fig. 5.5), whilst a $\lambda < 0$ implies an upward sloping ATM volatility term structure, and
- θ controls the short term ATM curvature.
- θ is a term that represents the difference between the current at-the-money volatility σ_0 and the long term at-the-money volatility ω , and
- λ is a parameter defined such that λ/τ is the mean reversion speed useful for ATM calendar spreads.



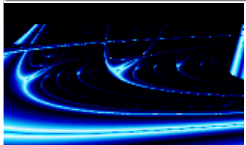
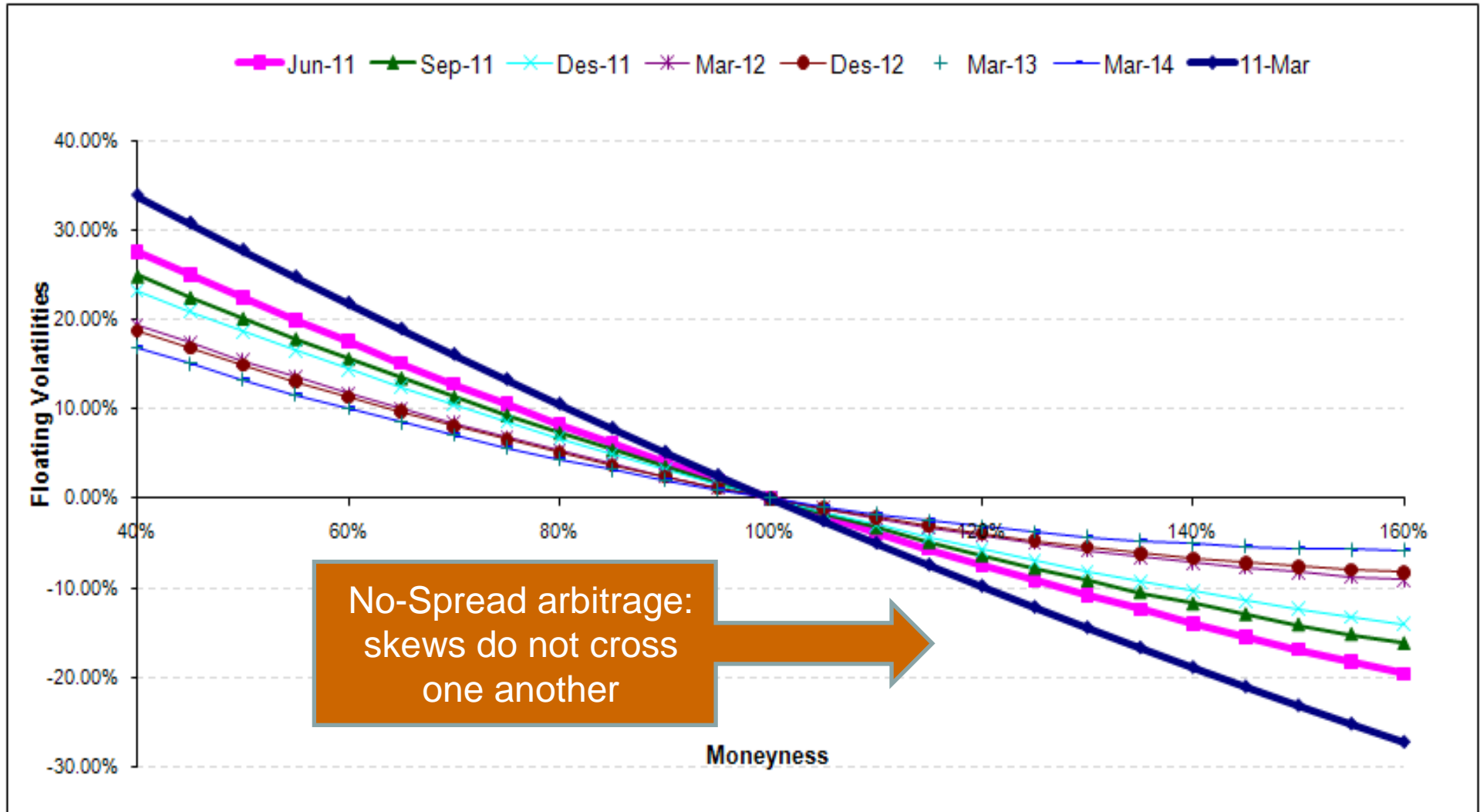
ATM Term Structure: 8 Feb 2011



Note: used to be mostly downward sloping before 2008 crisis

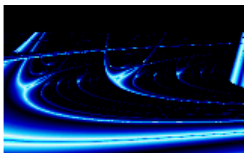


Volatility Surface: 8 Feb 2011



Optimisation

- Use all trades of previous 5 trading days
- Currently between 20 and 80 trades per day
- Usually have enough trades for first 3 expiries
- Optimisation with constraints can be a difficult road to trod by. Use Nelder-Mead algorithm
- Model error usually below 1.5%



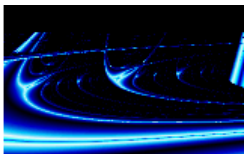
First Optimisation

The minimization problem we have to solve at time t_0 is stated as

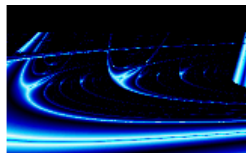
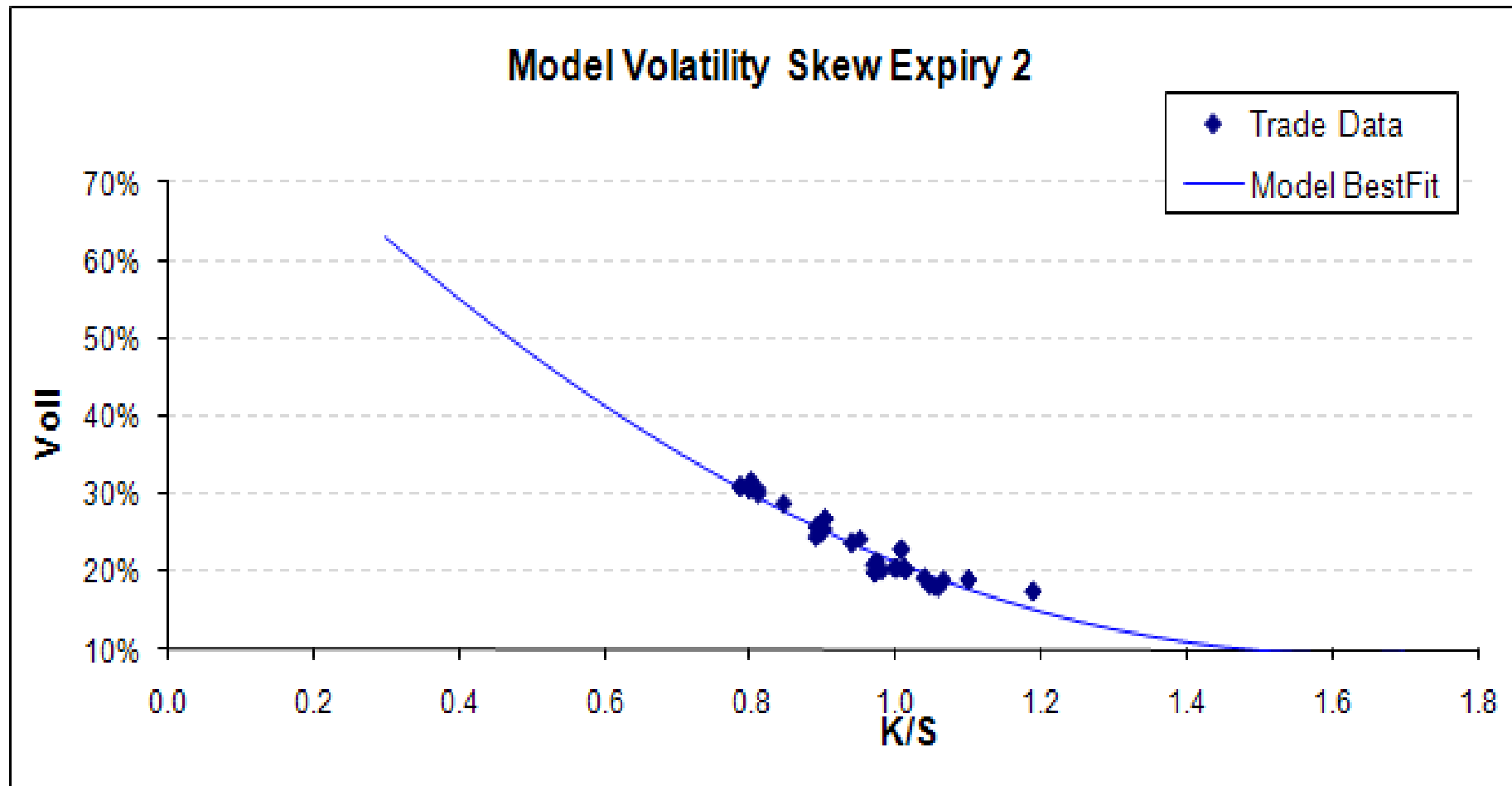
$$\omega_i \min_{\beta_k} \left\| \sigma_{t_i}^{model} - \sigma_{t_i}^{traded} \right\|^2 \quad \text{with } t_i \in [t_0 - h, t_0]; k = 1, 2, 3 \quad (7)$$

subject to the constraints; $\beta_0, \beta_2 > 0$, and $-1 < \beta_1 < 0$. Here $h \sim 7$ and ω_i are weights such that the optimisation are biased towards the most recent traded data.

- Model = quadratic functional form
- Enough data such that RMSE usually small
- Let's see how it's done..... remember constraints.....



Scatter Graph with Fitted Quadratic



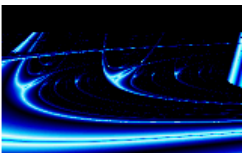
Model Error	1.171%
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ATM Vols

- ATM vols obtained from (3)

$$\sigma_{atm}^{model}(\beta_0, \beta_1, \beta_2, \tau) = \beta_0 + \beta_1 + \beta_2.$$

$$\begin{aligned}\sigma_{float}^{model}(\tau) &= \sigma_{model} - \sigma_{atm}^{model} \\ &= \beta_1(K - 1) + \beta_2(K^2 - 1).\end{aligned}\tag{9}$$

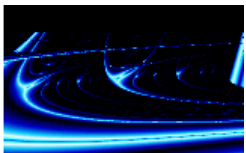


Second Optimisation

Eq. (9) gives us the floating volatilities only whilst Eq. (8) gives us the model ATM volatilities. To obtain the correct absolute volatilities, we need the correct ATM volatilities. This is now done by separately calibrating Eq. (4). As a basis for the at-the-money volatility term structure determination, we utilize the at-the-money volatilities defined by the optimised skews and given in Eq. (8). We then minimise the function

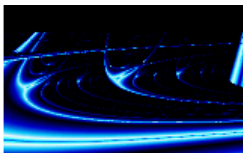
$$\min_{\theta, \lambda} \left\| \sigma_{atm}^{model}(\tau) - \frac{\theta}{\tau^\lambda} \right\|^2 \quad (10)$$

- Need data in at least 3 expiries to obtain curvature
- Even if we do not have any trades for further out expiries, we can now obtain model skews for these expiries
- Let's see how it's done.....



8 Feb 2011: parameters – strike spread

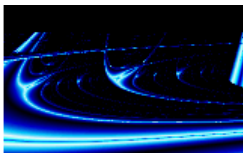
Expiry	Traded Average Moneyness		Traded Puts and Calls	
	Minimum	Maximum	Puts	Calls
Mar-11	82.63%	105.21%	20	23
Jun-11	78.59%	118.98%	27	25
Sep-11	62.95%	113.75%	15	4
Des-11	68.63%	91.23%	7	
Mar-12	85.63%	95.42%		6
Des-12				
Mar-13				
Mar-14				
Des-14				



8 Feb 2011: parameters – model

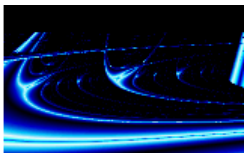
Safex Quadratic Deterministic Model Parameters					SABR Model Parameters		
					Current: 08-FEB-2011		
Expiry	Slope (ρ)	VolVol (v)	Shift (μ)	ATM	Beta	Rho	Volvol
Mar-11	-102.3%	28.193396%	93.971234%	Decay (λ)	80%	-6.126%	99.000%
Jun-11	-73.7%	18.909232%	77.443385%	Curvature (θ)	80%	20.106%	80.151%
Sep-11	-63.7%	15.823812%	71.042576%		80%	-97.665%	69.012%
Des-11	-58.0%	14.120959%	67.231423%		80%	-95.046%	63.274%
Mar-12	-54.1%	12.977843%	64.538312%		80%	-92.427%	59.357%
Des-12	-46.9%	10.906854%	59.327033%		80%	-85.209%	52.147%
Mar-13	-45.3%	10.471130%	58.167165%		80%	-83.140%	50.638%
Mar-14	-45.3%	10.471130%	58.167165%		80%	-75.645%	46.411%
Des-14	-45.3%	10.471130%	58.167165%		80%	-75.343%	47.312%

- Model error for quadratic lower than that for SABR
- Equivalent SABR parameters given due to some market players using the SABR model



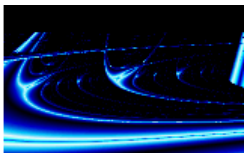
Model Robustness

- It is a well-known fact that volatility surfaces generated from models need to be stable in order to achieve reliable valuations and sound risk management calculations. In this light, the deterministic volatility model parameters are constrained and it can be shown that these parameters are unique.
- If a model volatility skew is generated on a daily basis, one runs the risk that the **wings can vibrate**. This is a consequence of the optimisation procedures employed and is obviously not how the markets trade on a daily basis. **In order to overcome this problem, an exponentially weighted moving average is used over the last couple of days.**



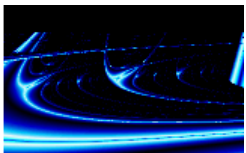
Conclusion

- The fitting methodology can be seen as a **best-decency fit** similar to fitting a yield curve to bond data.
- This means that we might not be able to price all options back to their original premiums but the skews are smooth and still **arbitrage free**.
- Found shape of skews to be fairly stable – similar to Tompkins
- Few human interventions are necessary during the process



Enhancements: further work

- Wings
- Traded data shows our market does not trade far away from the ATM: strike spread limited
- Skews published from 70% - 130%. Usually have to extrapolate
- Var futures (swaps) need more accurate wings
- Var futures valued from 50% - 150% in moneyness
- Interpolation needs reform – currently linear in volatility and strike (all options)
- Need to interpolate variance in $\text{Ln}(\text{strike})$
- Skews for illiquids – busy with research project together with Wits to shed some light onto problem



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