# Stock Price Volatility: a primer

FINANCIAL CHAOS THEORY

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*A nucleus of knowledge, experience and creativity in derivative products*

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**Abstract**
A price series or an economic indicator that changes a lot and swings wildly is said to be “volatile”. This simple and intuitive concept is the cause of many difficulties in finance. Unlike many other market parameters which can be directly observed, volatility has to be estimated. This is difficult, if not impossible, because we cannot say that volatility is necessarily stochastic or that it conforms to any mathematical model; all we know is that the evolution of volatility is uncertain. An accurate estimate of volatility is, however, crucial in many applications, including risk measurement and management as well as option pricing and hedging.

# 1 Introduction

The publication of the preference-free option pricing formula by *Fischer Black* and *Myron Scholes* in 1973 was a giant step forward in financial economics. Since then option pricing theory has developed into a standard tool for designing, pricing and hedging derivative securities of all types.

The *Black & Scholes* formula for pricing vanilla European options in an ideal market needs six inputs: the current stock price, the strike price, the time to expiry, the riskfree interest rate, the dividends and the volatility. Of these, the first three are known from the outset and the last three must be estimated. *Black & Scholes* assumed a perfect world in their analysis and took the last three as constants. In the real world, however, the correct values for these parameters are only known when the option expires. This means that the future values of these quantities need to be determined if an option is to be priced correctly.

The most important of the three uncertain parameters, is the volatility. Changes in interest rates (especially in a low interest rate environment) do not influence the price of an option as much as changes in volatility. Most options (and especially warrants) are short dated (expiries less than 12 months). The impact of small dividends[1] on the value of an option that is short dated is minimal. Dividend risk can be minimised through good research whereby the future dividends that will be paid during the next 12 to 18 months can be estimated quite accurately.

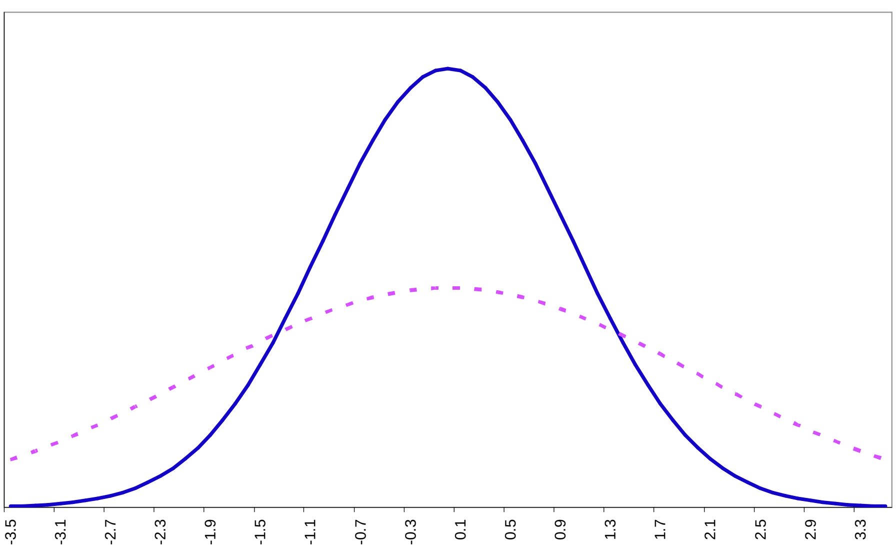
The importance of the volatility parameter was highlighted by *Black & Scholes* through their model. Practitioners now needed to estimate only one parameter, the volatility, and input it into a relative simple formula to find the price of an option. Of the three uncertain parameters, changing volatility has the biggest impact on the price of an option.

Volatility measures variability, or dispersion about a central tendency — it is simply a measure of the degree of price movement in a stock, futures contract or any other market. Volatility also has many subtleties that make it challenging to analyze and implement. The following question immediately comes to mind: can we estimate this seemingly complex quantity called volatility?

In this short note we'll explore 2 different ways to estimate volatility. Both of these are quite simple to implement in Microsoft Excel.

# 2 The Statistical Nature of Volatility

*Black & Scholes* assumed that financial asset prices are random variables that are lognormally distributed. Therefore, returns to financial assets, the relative price changes are usually measured by taking the differences between the logarithmic prices. These differences (the so-called log-relatives) are normally distributed. A normal distribution is indicated by a bell shaped curve. This is shown in Fig. 1.




[1]In SA most of the top 60 companies pay relatively small dividends.

**Figure 1**: The bell shaped normal cumulative distribution.

What does this all means in practise? Stock prices are usually observed at fixed intervals of time (daily, weekly or monthly) and we then have a time series of data. The logrelative returns are mathematically defined by the equation

$$u_i = \ln(S_i) - \ln(S_{i-1}) = \ln\left(\frac{S_i}{S_{i-1}}\right) \qquad (1)$$

where $S_i$ is the stock price at the end of the $i$-th interval and $\ln(\,)$ is the natural logarithmic function. We also assume that there are $n$ stock prices in our sample. This equation can easily be implemented in Microsoft Excel. This is illustrated in Fig. 2 where we used a few MTN share prices.

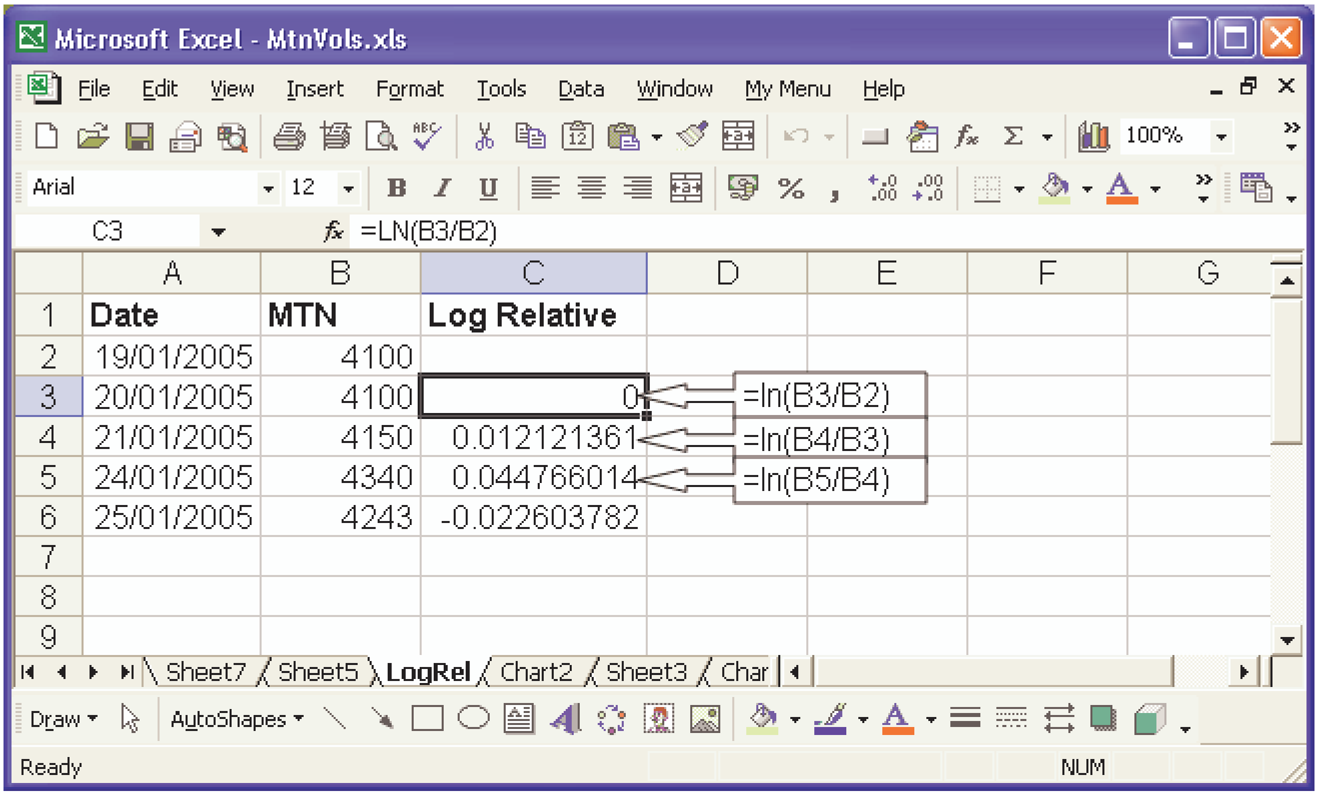



**Figure 2**: MTN logrelative prices. Also shown are the formulas as used in Excel.

Volatility is defined as the variation or dispersion or deviation of an asset's returns from their mean. In Fig. 1 we show two normal curves. Both have the same mean but the dotted line shows a greater dispersion than the continuous line. These two curves also illustrate that volatility indicates the range of a return's movement. Large values of volatility mean that returns fluctuate in a wide range – large risk. The most common measure of dispersion is the standard deviation of a random variable.

But, what does this all means? If we assume the mean of the logrelative returns is zero, then, a 10% volatility represents the following: in one year, returns will be within [-10%; +10%] with a probability of 68.3% (1 standard deviation from the mean); within [-20%; +20%] with a probability of 95.4% (2 standard deviations), and within [-30%; +30%] with a probability of 99.7% (3 standard deviations) — according to a normal distribution.

# 3 The Variance Rate of Return

In their paper in 1973, *Black & Scholes* mentioned the parameter $\sigma^2$ which they said was the "variance rate of the return" on the stock prices. *Black & Scholes* took this as a known parameter that is constant through the life of the option. Did they really know what this parameter was?

In a paper prior to their seminal one, *Black & Scholes* gave more insight into the variance rate of return. There they stated that they estimated the instantaneous variance from the historical series of daily stock prices. They thus defined volatility as the amount of variability in the returns of the underlying asset. *Black & Scholes* determined what is today known as the historical volatility and used that as a proxy for the expected volatility in the future. In that paper they tested several implications of their model empirically by using a sample of 2 039 calls and 3 052 straddles traded on the New York stock exchange between 1966 and 1969.

In analyzing their results they noted that the variance actually employed by the market is too narrow and that the historical estimates of the variance include an attenuation bias, i.e., the spread of the estimates is greater than the spread of the true variance. This implies that for securities with a relatively high variance, the market prices underestimate the variance, while using historical price series would overestimate the variance and the resulting *Black & Scholes* model price would thus be too high; the converse is true for relative low variance securities. Was this the first observation of a volatility skew or smile? In further tests *Black & Scholes* found that their model performed very well when the true variance rate of the stock was known.

# 4 Estimation of Volatility

## 4.1 Trading or Nontrading Days

To estimate the volatility of a stock price empirically, the stock price is usually observed at fixed intervals of time. These intervals can be days, weeks or months[2] . Before any calculation can be done, however, a question one needs to answer is whether the volatility of an exchange-traded instrument is the same when the exchange is open as when it is closed.

Some people argue that information arrives even when an exchange is closed and this should influence the price. A lot of empirical studies have been done and researchers found that volatility is far larger when the exchange is open than when it is closed. The consequence of this is that if daily data are used to measure volatility, the results suggest that days when the exchange is closed should be ignored.

## 4.2 Historical Volatility

The historical volatility is the volatility of a series of stock prices where we look back over the historical price path of the particular stock. We previously mentioned that the most common measure of dispersion is the standard deviation. The historical volatility estimate is thus given by

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} \qquad (2)$$

where $\bar{u}$ is the mean defined by

$$\bar{u} = \frac{1}{n}\sum_{j=1}^{n} u_j .$$

[2]One has to be consistent; if the frequency of observation is every Thursday at midnight, the returns all need to correspond to such a period

$u_i$ was defined in Equation (1). $\sigma$ in Equation (2) gives the estimated volatility per interval. To enable us to compare volatilities for different interval lengths we usually express volatility in annual terms. To do this we scale this estimate with an annualization factor (normalising constant) $h$ which is the number of intervals per annum such that

$$\sigma_{an} = \sigma * \sqrt{h}.$$

If daily data is used the interval is one trading day and we use $h = 252$, if the interval is a week, $h = 52$ and $h = 12$ for monthly data[3] .

Equation (2) is just the standard deviation of the sampled series $u_j$. Fig. 3 shows how this can be implemented in Microsoft Excel where we show the daily closing values for MTN from 1 November 2004 till 25 January 2005. In Fig. 4 we plot the 3 month historical volatility for MTN.

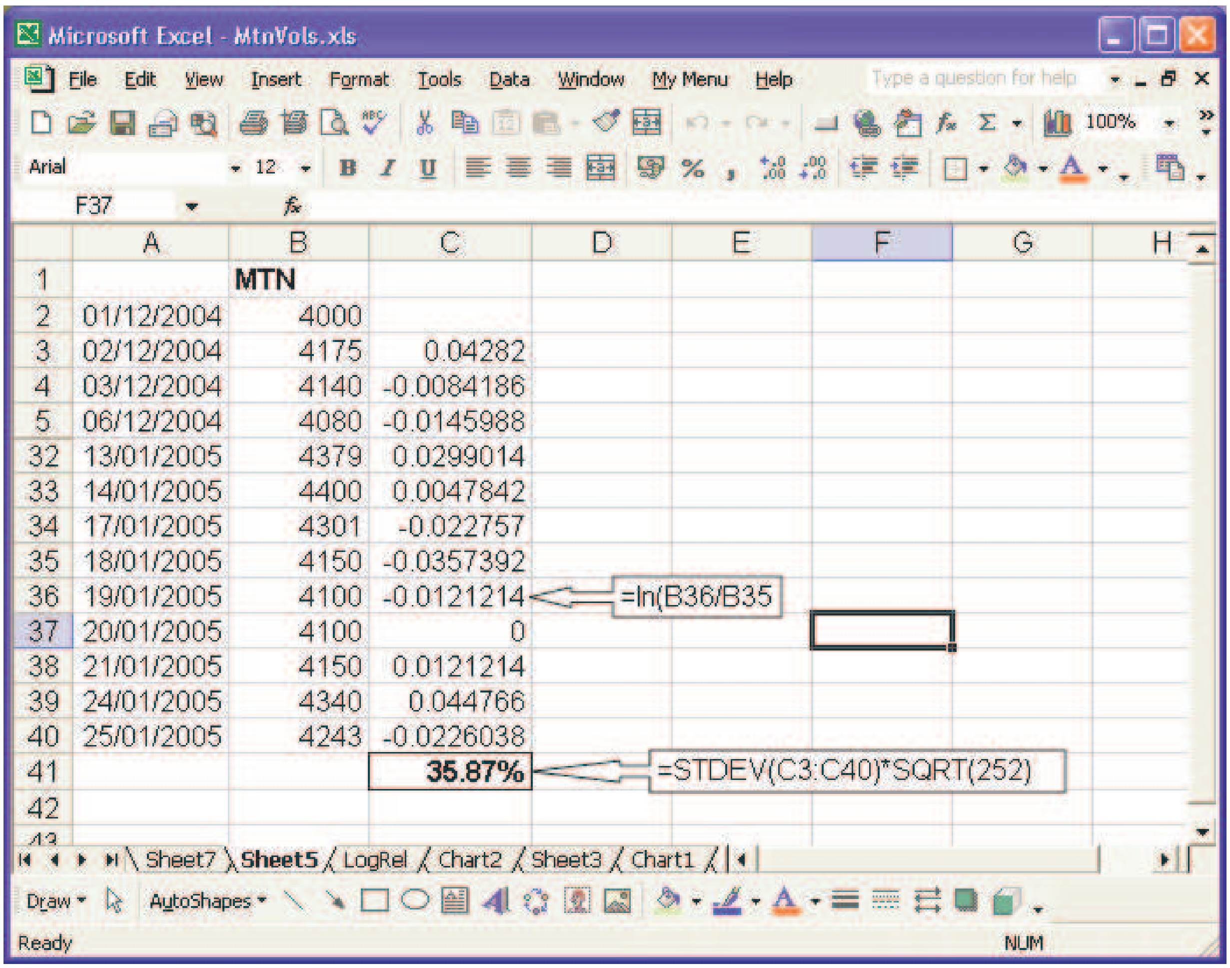



**Figure 3**: Historical volatility: Excel implementation.

[3]There is approximately 252 trading days per annum

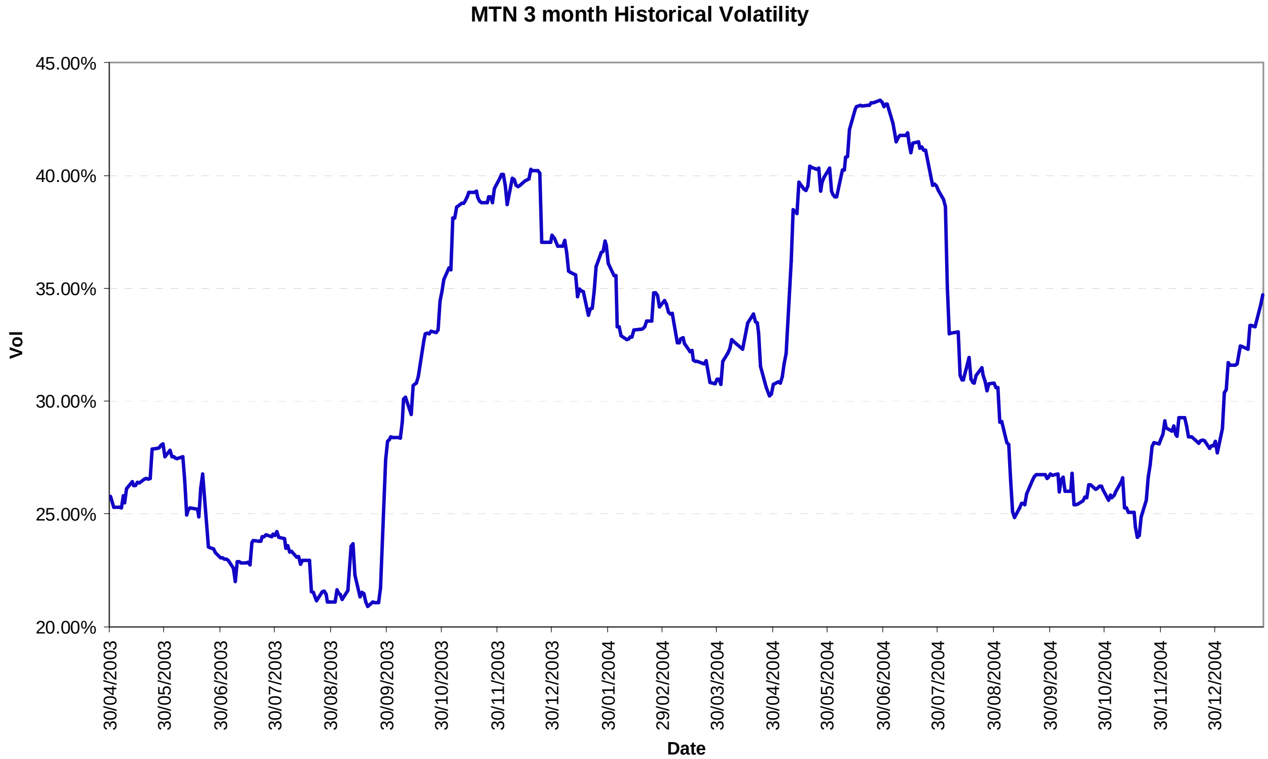



**Figure 4**: Moving 3 month historical volatility for MTN from February 2003.

## 4.3 Implied Volatility

A simple option pricing model (like the *Black & Scholes* model) will give a theoretical price for an option as a function of the implicit parameters — constant volatility being one. However, if the option is traded, the market price might not be the same as the model price. In that case one might ask, which volatility estimate does one have to use in the model so that the model price and the market price are the same? This is the implied volatility. In a constant volatility framework, implied volatility is the volatility of the underlying asset price that is implicit in the market price of an option according to a particular model.

We illustrate the basic idea by analysing the MTNABA warrant. This warrant has a strike price of R40, it expires on 17 March 2005 and the cover ratio is 10. Fig. 5 shows the MTN and MTNABA prices from February 2004. The warrant price follows the MTN price but due to the gearing of the warrant the swings can be wilder.

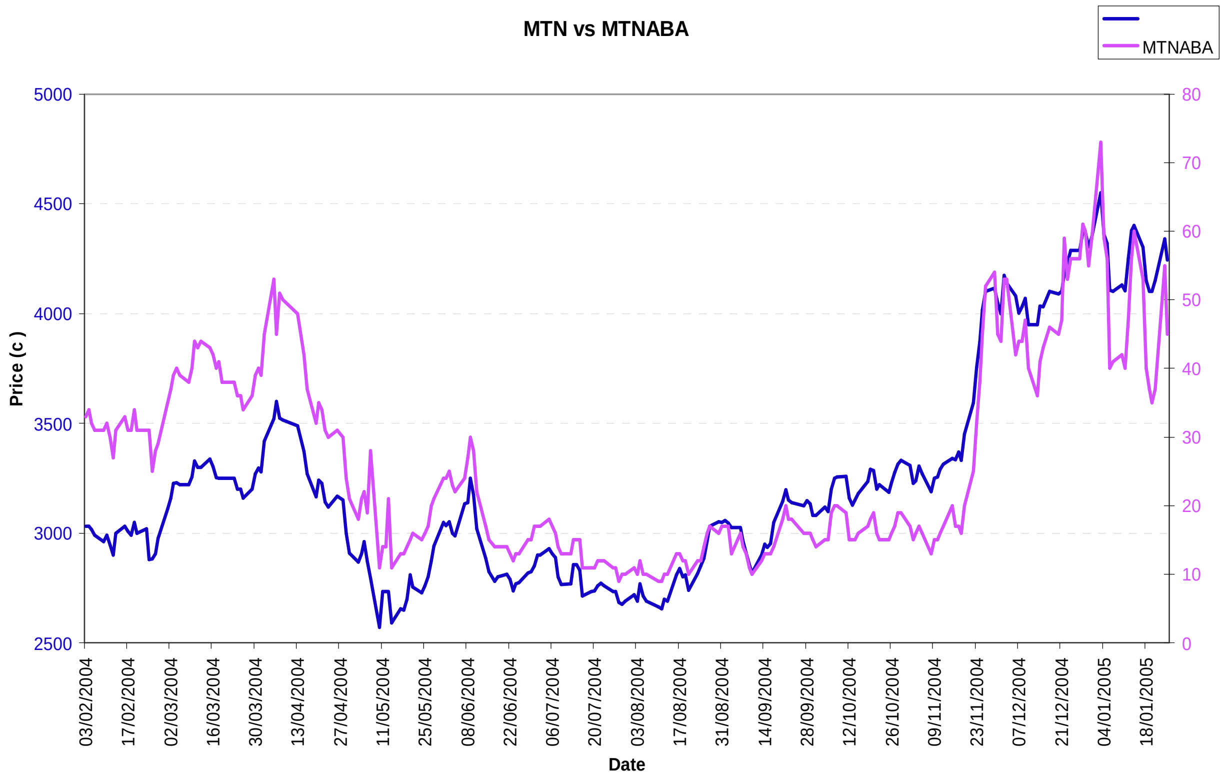



**Figure 5**: MTN and MTNABA price behaviour.

To calculate the implied volatility we ask ourselves: on 1 December 2004, the warrant price was R0.44 and MTN's price was R40 (the same as the strike price), we now want to know, if we substitute this price (44 cents), into the *Black & Scholes* equation, what volatility will pop out!

Before we can do anything, we need to know the parameters mentioned in the Introduction. Current interest rates are at 8.5% and MTN's dividend yield is 1%.

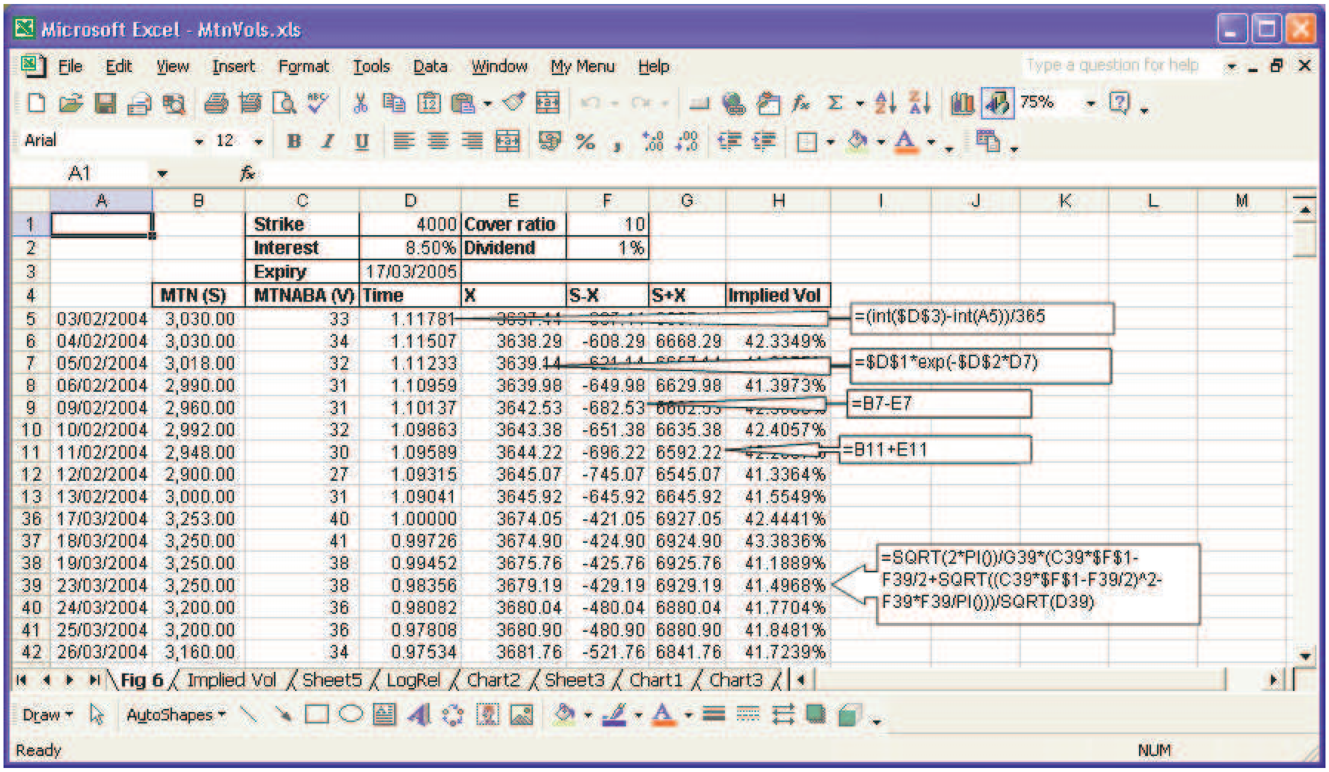
If you have an option pricing spreadsheet, you can substitute all the parameters into that and use Excel's Goalseek to search for the volatility[4] . If you do not have such a spreadsheet you can use the formula due to *Corrado* and *Miller*. They refer to it as the improved quadratic formula where

$$\sigma\sqrt{T} = \frac{\sqrt{2\pi}}{S' + X}\left[V - \frac{S' - X}{2} + \sqrt{\left(V - \frac{S' - X}{2}\right)^2 - \frac{(S' - X)^2}{\pi}}\right].$$

Here $X = K\,e^{-rT}$ which is the discounted strike price, $S' = S\,e^{-dT}$ where $S$ is the stock price, $K$ the strike price, $V$ is the warrant price multiplied by the cover ratio, $r$ is the risk-free interest rate, $d$ is the dividend yield, $\pi = 3.14159265\ldots$ (Archimedes' constant) and $T$ is the time to expiry. It is accurate over a wide range of strike prices.

Fig. 6 shows an implementation in Excel (ensure that the sheet is set up as shown with all the parameters in the cells as shown). We show the implied volatilities calculated for a few MTNABA warrant prices. In Fig. 7 we plot the MTNABA warrant price and the implied volatility time series.

[4]Remember to multiply the warrant price by the cover ratio.




| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | | | Strike | 4000 | Cover ratio | 10 | | |
| 2 | | | Interest | 8.50% | Dividend | 1% | | |
| 3 | | | Expiry | 17/03/2005 | | | | |
| 4 | | MTN (S) | MTNABA (V) | Time | X | S-X | S+X | Implied Vol |
| 5 | 03/02/2004 | 3,030.00 | 33 | 1.11781 | [illegible] | [illegible] | [illegible] | [illegible] |
| 6 | 04/02/2004 | 3,030.00 | 34 | 1.11507 | 3638.29 | -608.29 | 6668.29 | 42.3349% |
| 7 | 05/02/2004 | 3,018.00 | 32 | 1.11233 | 3639.14 | [illegible] | [illegible] | [illegible] |
| 8 | 06/02/2004 | 2,990.00 | 31 | 1.10959 | 3639.98 | -649.98 | 6629.98 | 41.3973% |
| 9 | 09/02/2004 | 2,960.00 | 31 | 1.10137 | 3642.53 | -682.53 | [illegible] | [illegible] |
| 10 | 10/02/2004 | 2,992.00 | 32 | 1.09863 | 3643.38 | -651.38 | 6635.38 | 42.4057% |
| 11 | 11/02/2004 | 2,948.00 | 30 | 1.09589 | 3644.22 | -696.22 | 6592.22 | [illegible] |
| 12 | 12/02/2004 | 2,900.00 | 27 | 1.09315 | 3645.07 | -745.07 | 6545.07 | 41.3364% |
| 13 | 13/02/2004 | 3,000.00 | 31 | 1.09041 | 3645.92 | -645.92 | 6645.92 | 41.5549% |
| 36 | 17/03/2004 | 3,253.00 | 40 | 1.00000 | 3674.05 | -421.05 | 6927.05 | 42.4441% |
| 37 | 18/03/2004 | 3,250.00 | 41 | 0.99726 | 3674.90 | -424.90 | 6924.90 | 43.3836% |
| 38 | 19/03/2004 | 3,250.00 | 38 | 0.99452 | 3675.76 | -425.76 | 6925.76 | 41.1889% |
| 39 | 23/03/2004 | 3,250.00 | 38 | 0.98356 | 3679.19 | -429.19 | 6929.19 | 41.4968% |
| 40 | 24/03/2004 | 3,200.00 | 36 | 0.98082 | 3680.04 | -480.04 | 6880.04 | 41.7704% |
| 41 | 25/03/2004 | 3,200.00 | 36 | 0.97808 | 3680.90 | -480.90 | 6880.90 | 41.8481% |
| 42 | 26/03/2004 | 3,160.00 | 34 | 0.97534 | 3681.76 | -521.76 | 6841.76 | 41.7239% |

**Figure 6**: Implied volatility: implementation in Excel.

If we had many warrants, which vary in strike price and time to expiration, that were written on the same underlying like MTN, we would observe a term structure of volatilities and a volatility "smile" or "skew". This is due to systematic deviations from the predictions of the *Black & Scholes* model and warrants another broader discussion.

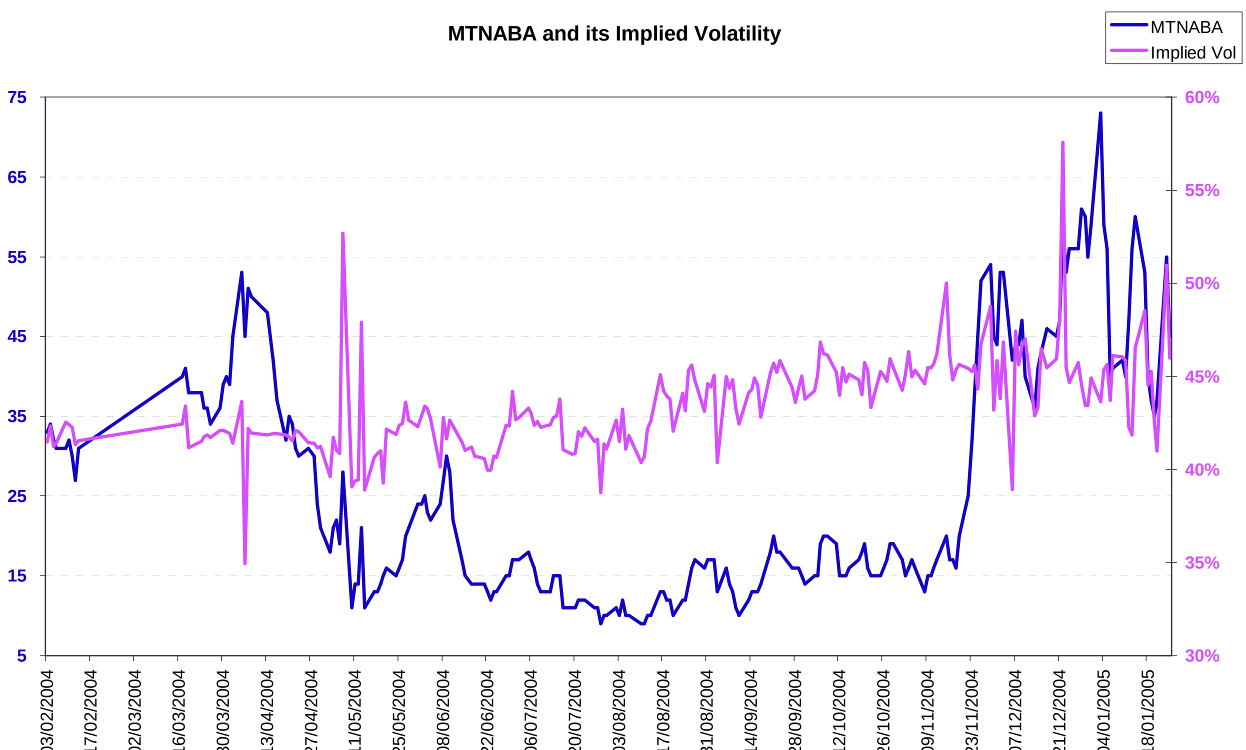



**Figure 7**: MTNABA warrant price and implied volatility.

# 5 Difference between Implied and Statistical Volatilities

Implied volatilities should be viewed differently from statistical volatilities even though they both forecast the volatility of the underlying asset over the life of the option. The two forecasts differ because they use different data and different models. Implied methods use current data on market prices of options, so the implied volatility contains all the forward expectations of investors about the likely future price path of the underlying. Also, due to the *Black & Scholes* assumptions this method assumes that the underlying's price path is continuous.

Contrast this with statistical methods which use historic data on the underlying asset returns in a discrete time model for the variance of a time series.

# 6 Realized/Actual Volatility

This is the historical volatility calculated looking “backward" when an option has expired. As an example, let's say a trader wants to write an option today that expires in 3 months time. To estimate the volatility he/she might calculate the historical volatility of the past 3 months. If similar options are trading in the market he/she might calculate the implied volatility. The actual volatility will, however, only be known at expiry. Once the 3 months have passed, one can calculate the realized volatility (actual variance) between the original trade date and expiry because the actual price path is then known.

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