Certain Uncertain Volatility Constantly

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Abstract

A price series or an economic indicator that changes a lot and swings wildly is said to be "volatile". This simple and intuitive concept is the cause of many difficulties in finance. Unlike many other market parameters which can be directly observed, volatility has to be estimated. This is difficult, if not impossible, because we cannot say that volatility is necessarily stochastic or that it conforms to any mathematical model; all we know is that the evolution of volatility is uncertain. An accurate estimate of volatility is, however, crucial in many applications, including risk measurement and management as well as option pricing and hedging.

The effort to understand the universe is one of the very few things that lifts human life a little above the level of farce, and gives it some of the grace of tragedy.

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1 Introduction

The publication of the preference-free option pricing formula by *Fischer Black* and *Myron Scholes* in 1973 was a giant step forward in financial economics [BS 73]. Since then option pricing theory has developed into a standard tool for designing, pricing and hedging derivative securities of all types. The *Black & Scholes* formula for pricing vanilla European options in an ideal market needs six inputs: the current stock price, the strike price, the time to expiry, the riskfree interest rate, the dividends and the volatility. Of these, the first three are known from the outset and the last three must be estimated. *Black & Scholes* assumed a perfect world in their analysis and took the last three as constants. In the real world, however, the correct values for these parameters are only known when the option expires. This means that the future values of these quantities need to be determined if an option is to be priced correctly [Wi 00].

The most important of the three uncertain parameters, is the volatility[1]. *Black & Scholes* highlighted this through their model. They, however, knew that volatility changes over time. As far back as 1976, *Fischer Black* wrote

> "Suppose we use the standard deviation ... of possible future returns on a stock ... as a measure of volatility. Is it reasonable to take that volatility as constant over time? I think not."

Figure 1 shows a graph of the annual volatility for the Alsi100 using monthly data. It is clear that volatility is not constant.

But, what is this "volatility"? As a concept, volatility seems to be simple and intuitive. It measures variability, or dispersion about a central tendency but, volatility has many subtleties that make it challenging to analyze and implement. The following questions immediately comes to mind: is volatility a simple intuitive concept or is it complex in nature, what causes volatility, how do we estimate volatility and can it be managed? In this paper we will try to shed some light on these and many more questions. We will concentrate on the practical estimation of volatility and how different measures thereof can help the trader and risk manager to understand and handle changing volatility.

We start this paper by giving some background information in the first two sections. Here we stress the importance of volatility and take a quick tour through the development and intuitions behind the *Black & Scholes* formula. After these introductory notes we discuss some techniques to estimate the volatility in Section 4. We describe the historical volatility, implied volatility and some Extreme-Value methods. After these rather theoretical discussions, we get practical in Section 5 and start asking pertinent questions every trader and risk manager should ask about volatility. We start to answer them in Section 6 by showing that volatility is not constant over

[1] Changes in dividends and interest rates do not influence the price of an option as much as changes in the volatility.

time. In Section 7 we give some guidelines in choosing a sample size in determining the historical volatility. In Section 8 we give some useful ways in which the Extreme-Value methods can be used in conjunction with historical volatility in making trading decisions. In the last part, Section 9, we show how tick data (high frequency data) can be used to understand the intraday dynamics of the market.

Note that we only concern ourselves with the equities market, although, most of the ideas could be implemented in the foreign exchange and interest rate markets as well.

2 Importance of Volatility

We know that volatility has been viewed as natural for a long time, certainly predating any sophistication in capital markets. At the beginning of the 21st century, almost every financial decision there is to make is interesting because of volatility [Ne 97]. Volatility holds a certain status in the finance industry and this is enhanced when prominent people talk about it. Every person in the markets takes note when Alan Greenspan, the governor of the Federal Reserve Bank in the USA, speaks. He often refers to the volatility in the financial markets.

Since 1973 the array of available and actively traded products have expanded enormously. Some derivative instruments are extremely complex to value and understand. But, for complex or simple derivative instruments one thing is central: if it contains optionality its valuation model requires at least one volatility parameter. Volatility is thus important to a trader as well as to a risk manager who needs to understand the risks associated with these instruments. People have also generalized the *Black & Scholes* model where the volatility is taken as a stochastic parameter [Le 00]. But, *this does not solve the problem of estimation* since, for these models, the user has to specify a set of parameters to define a time-varying stochastic volatility process. This can be difficult.

Any person concerned with the financial markets knows the importance of volatility. Figure 2 shows a graph of the DJIA between May 1999 and July 2001. The Dow has moved sideways - but definitely not in a straight line. Even the layman will tell you there are large swings and this looks volatile. Whether you are an equities broker, derivatives trader or risk manager, events like that on 11 September 2001 influence the markets, volatility and thus your life!

3 The Black-Scholes Option Pricing Formula

A common misconception about option pricing models is that their typical use by option traders is to indicate the "right" price of an option. Options pricing models do not necessarily give the "right" price of an option. The "right" price is what someone is willing to pay for a particular option. An efficient market will give the best and truest prices for options. The true benefits of an option pricing model is that it provides an accurate "snap shot" of current market conditions.

A model is important and useful, however, to break the option's market price into each of the factors that comprise it i.e., for risk management purposes. A model can be used to examine each factor separately and assess its individual contribution to the determination of the option's price. Furthermore, by forecasting each of the individual factors, one can forecast an option's price over a wide range of different scenarios. In Appendix A we give a few general, but important, comments in working with models. In this section I will give a very brief overview of the *Black & Scholes* formula and its development.

3.1 A History of Option Trading

In April 1973, a major innovation in securities trading took place with the opening of the Chicago Board Options Exchange (CBOE). The CBOE began with call options on 16 heavily traded common stocks and has subsequently evolved into one of the largest exchanges in the world in terms of the value of securities traded. This was the first organized trading of options on an exchange. Over the counter (OTC) options have a much longer history. It started with the Tulip-bulb craze in Holland in the early seventeenth century [Ma 90].

Options on equities have been traded on the Johannesburg Stock Exchange (JSE) since the end of the last century. The first standardized option contract in South Africa was written in 1987 on E168 stock of Eskom (this still was an OTC option). This led to the formation of the South African Futures Exchange (SAFEX) in September 1988 where standardized option contracts on equity and interest rate futures contracts can be traded. Some more general comments on the South African option market is given in Appendix B.

3.2 A History of Option Valuation

The previous section showed that options have been traded for many centuries without a simple way of determining the price. The first quantitative research into option pricing was done by *Bachalier*. *Bachelier* wrote his doctoral dissertation in 1900 and priced options with an arithmetic Brownian process. This process implies a normally distributed stock price behaviour. He also assumes that the mean expected price change per unit time is zero [Sm 76]. *Bachelier* views the stock market as a fair gamble; he feels that competition will reduce the expected return to zero.

Since Bachelier people realized that stock prices are better described by a lognormal distribution and their behaviour can be modeled by geometric Brownian motion[2]. Research into finding better statistical ways to describe the market is still continu-

[2]This roughly means that the market maintains a constant expected percentage move.

ing[3]. Many different distributions have been proposed like the parabolic distribution. Most of these lead to complex equations to solve and have not gone down well with practitioners.

It was only since the early sixties that people seriously tried to calculate option prices. *Sprenkle* (1961), *Ayres* (1963), *Boness* (1964), *Samuelson* (1965) and *Chen* (1970) all produced valuation formulas similar in form to the *Black & Scholes* model but with constants which were either difficult to calculate (or to define) or had to be obtained empirically [BS 73]. All these formulas had the *variance rate of return* on the stock prices as a parameter. *Black & Scholes* defined this in their paper as follows

> "the variance rate of return on a security is the limit, as the size of the interval of measurement goes to zero, of the variance of the return over that interval divided by the length of the interval."

3.3 The *Black & Scholes* Environment

The market is complex and to be able to obtain a useful model, one has to simplify the market and make some assumptions. These assumptions can later be relaxed to get a more realistic model. The following is a list of the more important assumptions *Black & Scholes* made in their analysis [MS 00]:

- *The underlying follows a lognormal random walk.* This was not a new assumption as was explained in the previous section.
- *We live in a risk-neutral world i.e., investors require no compensation for risk.* The expected return on all securities is thus the riskfree interest rate with the consequence that there are no arbitrage opportunities. This was one of *Black & Scholes* ' insights and is known as risk-neutral valuation.
- *Short selling of securities with full use of the proceeds is permitted.*
- *The risk-free interest is known, there are no dividends and no transaction costs.*
- *Delta hedging is done continuously.* This is impossible in a realistic market but makes their analysis possible.

3.4 The *Black & Scholes* Analysis

The form of the option's pricing formula was already known in the late sixties. These formulas, however, had two or more parameters that were hard to estimate. So what was the problem? Analytically, the payoff function of a European call is given by

$$V(T) = \max[0, S_T - K] \tag{1}$$

[3]Some evidence, however, suggests that financial markets can be better described by a mixed process: arithmetic Brownian motion in the short term and geometric Brownian motion in the long run.

with S_T and K the price of the underlying asset at expiry and the strike price respectively. To know what an option is worth one only has to know the stock price at expiry. Easy.....? Well, we all know, to forecast stock prices is very difficult if not impossible.

Black & Scholes subscribed to the idea that stock prices were described by geometric Brownian motion. Their breakthrough came when they realized that, when a hedged position for an option is in equilibrium, the expected return on such a hedge must be equal to the return on a riskless asset. This insight led them to construct a riskless hedge for an option. They showed that, of the three securities: the option, the underlying stock and a riskless money market security, any two could be used to exactly replicate the third by a trading strategy.

In their original paper [BS 73] *Black & Scholes* showed that they could replicate the money market security. They stated that if one was short one derivative of value f and long an amount x of the underlying stock S, the portfolio is worth Π such that

$$\Pi = -f + xS.$$

This replicating portfolio must also be self-financing, which means you neither consume from it nor add money to it beyond an initial loan or deposit Π [Le 00, HK 00]. Now, the change $\Delta\Pi$ in the portfolio during time Δt is given by

$$\Delta\Pi = -\Delta f + x\Delta S.$$

If we have a completely riskfree change $\Delta\Pi$ in the portfolio value Π then it must be the same as the growth we would get if we put the equivalent amount of cash in a riskfree interest-bearing account. This means that

$$\Delta\Pi = r\Pi\Delta t$$

where r is the riskless interest rate[4]. Mathematically this riskless hedging and return arguments imply a differential equation[5] relating the value of the option to the value of the underlying asset. In using stochastic calculus *Black & Scholes* derived it to be[6]

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2}{\partial S^2} V(S,t) + \frac{\partial}{\partial t} V(S,t) + (r-d)S\frac{\partial}{\partial S} V(S,t) = rV(S,t) \tag{2}$$

where $V(S,t)$ is the option value, S is the current spot stock price, d is the dividend yield and σ^2 is the variance rate of the return on the stock prices. To obtain Eq. (2) no assumption about the specific kind of option has been made. This partial differential equation is valid for both calls and puts. The chosen type will be obtained by selecting the appropriate boundary conditions. For a call this is given in Eq. (1).

[4]The return on the portfolio is equal to the return on a riskless asset.

[5]A differential equation is a mathematical description of some phenomenon, system or process. This mathematical model can be used to analyse and describe the phenomenon. Once it is solved, we can describe the future behaviour of the phenomenon.

[6]I here give Black's general equation that he derived in 1976 by including a continuous constant dividend yield d [Bl 76]. The original equation had $d = 0$.

3.5 The Seminal Formula

Another breakthrough for *Black & Scholes* was that they realised that they can transform the differential equation in Eq. (2) into the equation describing the heat transfer through a medium like a wall or glass. This is a well known equation in physics and was solved by physicists at the beginning of the nineteenth century[7]. This insight and the assumptions described in the previous sections led them to solve the differential equation in Eq. 2 and derive their pricing formula. We outline their methodology in Appendix C. In general form we get

$$V(S,t) = Se^{-d\tau}N(\phi x) - Ke^{-r\tau}N(\phi y) \tag{3}$$

where

$$\begin{aligned} x &= \ln\frac{S}{K} + (r - d + \frac{1}{2}\sigma^2)\tau \\ y &= x - \sigma\sqrt{\tau}. \end{aligned}$$

where ϕ is a binary parameter defined as

$$\phi = \begin{cases} 1 & \text{for a call option} \\ -1 & \text{for a put option.} \end{cases}$$

Here, τ is the time to maturity and $N(x)$ is the cumulative standard normal distribution function.

The beauty of this formula lies in the fact that one does not need to estimate market expectation or risk preferences. This was a revolutionary improvement over its predecessors. The only parameter that needs to be estimated is the variance or volatility. Note that the volatility needed in the *Black & Scholes* formula is the volatility of the underlying security that will be observed in the future time interval τ - *volatility thus needs to be predicted* [MS 00]. *Black & Scholes*, however, assumed that the variance is known and that it is constant.

3.6 Interpretation of the Solution

The solution to the *Black & Scholes* differential equation can be interpreted in a risk-neutral expectations manner (we describe the call): If we exercise the call at maturity we will receive the stock but will have to pay the strike price K. This exchange will not take place if the call does not finish in-the-money. The first term $S(t)N(x)$ can thus be seen as the weighted present value of receiving the stock (this is the stock's potential value and our potential benefit) if and only if $S_T > K$. The second term $-Ke^{-r\tau}N(y)$ is the weighted present value of paying the strike price (this is our potential cost or loss).

In summary we can say

[7] Heat or diffusion equations goes back to the beginning of the 19th century. They were successfully used to model smoke particles in air, flow of heat from one part of an object to another, chemical reactions, dispersions of populations and dispersions of pollutants in a running stream.

- V≡ difference between what you would earn from the stock at T and what you would pay for it (the strike price K) at T.

3.7 The Variance Rate of Return

In a paper prior to the seminal one, *Black & Scholes* gave more insight into the variance rate of return [BS 72]. There they stated that they estimated the instantaneous variance from the historical series of daily stock prices. They thus defined volatility as the amount of variability in the returns of the underlying asset. *Black & Scholes* determined what is today known as the historical volatility and used that as a proxy for the expected volatility in the future. In this paper they test several implications of their model empirically by using a sample of 2 039 calls and 3 052 straddles traded on the New York stock exchange between 1966 and 1969.

In analyzing their results they noted that the variance actually employed by the market is too narrow and that the historical estimates of the variance include an attenuation bias, i.e., the spread of the estimates is greater than the spread of the true variance. This would imply that for securities with a relatively high variance, the market prices would imply an underestimate in the variance, while using historical price series would overestimate the variance and the resulting *Black & Scholes* model price would thus be too high; the converse is true for relative low variance securities. In further tests *Black & Scholes* found that their model performed very well when the true variance rate of the stock was known.

3.8 Empirical Research

Since *Black & Scholes* published their formula, a lot of empirical research has been undertaken to compare the *Black & Scholes* model price of options to market prices [Ro 87]. Much of this research focused on the volatility because this is the only parameter that needs to be estimated. People started to realize that historical and implied volatility changes through time. They also observed the implied variance rate declines as the exercise price increases [MM 79]. People thus observed a volatility skew. This then started the quest to understand volatility better and the search for better estimates thereof. This is described next.

4 Estimating Volatility

In Sec. 3 we saw that the volatility parameter used in the *Black & Scholes* equation is the future volatility i.e., the volatility from the transaction date to the expiry date. This is not known and volatility thus needs to be estimated or predicted .

In South Africa, OTC and Safex options are quoted on volatility. Traders might think that because volatility is a market "price" it is set by the market like the price of a share. However, how do you as an option buyer knows that this is the correct volatility? How did the writer of the option estimate the volatility in the first place especially if it is an OTC option? There are many variations in the method of measurement [Ta 97]. In this section we describe a few methods to estimate volatility. We will keep it simple and will not wonder into the complex field of the ARCH[8] or regression models.

4.1 Trading or Nontrading Days

To estimate the volatility of a stock price empirically, the stock price is usually observed at fixed intervals of time. These intervals can be days, weeks or months[9]. Before any calculation can be done, however, a question one needs to answer is whether the volatility of an exchange-traded instrument is the same when the exchange is open as when it is closed.

Some people argue that information arrives even when an exchange is closed and this should influence the price. A lot of empirical studies have been done and researchers found that volatility is far larger when the exchange is open than when it is closed [Hu 97]. The consequence of this is that if daily data are used to measure volatility, the results suggest that days when the exchange is closed should be ignored.

4.2 Historical Volatility

Let S_i be the stock price at the end of the ith interval and let

$$u_i = \ln \frac{S_i}{S_{i-1}} \tag{4}$$

be the log relative prices[10] for $i = 1, 2, \ldots, n$ with n the number of returns in the historical sample [Hu 97]. The historical volatility estimate is then given by

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (u_i - \overline{u})^2} \tag{5}$$

where $\overline{u}$ is the mean defined by

$$\overline{u} = \frac{1}{n} \sum_{j=1}^{n} u_j .$$

σ in Equation (5) gives the estimated volatility per interval. To annualize the volatility, we need to scale this estimate with an annualization factor h which is the number

[8] Auto Regressive Conditional Heteroskedasticity.

[9] One has to be consistent; if the frequency of observation is every Thursday at midnight, the returns all need to correspond to such a period

[10] Remember, the assumption is that stock prices are lognormally distributed. Their behaviour are described by geometric Brownian motion.

of intervals per annum such that[11]

$$\sigma_{an} = \sigma * \sqrt{h}.$$

If daily data is used the interval is one trading day and we use $h \approx 252$, if the interval is a week, $h = 52$ and $h = 12$ for monthly data[12].

Equation (5) is just the standard deviation[13] of the sampled series u_i. Table 1 provides an example of historical volatility calculation. It shows daily prices of Anglo American (AGL) in succession for 1 month (July 2001) i.e., 22 trading days. This yields 21 returns. The average for the log relatives u_j is -0.0055695 and the standard deviation is 0.0246051. This gives an annualized historical volatility of 39.059%.

4.3 Noncentered Volatility

In Table 1 we see that the mean return $\overline{u}$ is close to zero (especially for daily data). This is in general the case. So, what would happen if we discard it? As a bonus to very busy option traders, it means there is one parameter less to estimate. Equation (5) is then turned into the (nonweighted) noncentered volatility σ' that can be expressed as

$$\sigma' = \sqrt{\frac{1}{n}\sum_{j=1}^{n} u_j^2} \tag{6}$$

Another reason for ditching the mean return is that it makes the estimation of volatility closer to what would affect a trader's P/L. To illustrate this suppose we use a week's worth of returns to calculate the volatility and let's assume that each day was down 2%. If the mean were subtracted from each daily return, the volatility for the week would be zero. That certainly does not accord with a trader's expectation going into that week, and, the volatility the trader experienced during the week would be much higher than zero. The third reason is that zero-mean volatilities are better at forecasting future volatilities. In Table 1 we listed u_j^2 and calculated σ' to be 41.138% for Anglo during July 2001.

Figlewski gives another strong reason for using σ' instead of σ. He reasons that, since volatility is measured in terms of deviations from the mean return, an inaccurate estimate of the mean will reduce the accuracy of the volatility calculation [Fi 94]. This is especially true for short time series like 1 to 3 months (which are the time frames used by most option traders to estimate volatilities). He shows that one can have a standard deviation around the true mean of up to 85%. In other words, roughly one third of the time, the trader's volatility estimate will be calculated from a sample mean that is more than 85 percentage points above or below the correct value on an annualized basis.

[11] *Diebhold*, *Hickman*, *Inoue* and *Schuermann* caution in using a scaling of $\sqrt{h}$. They call this a "first generation rule of thumb" that can be improved [DH 96].

[12] There is approximately 252 trading days per annum

[13] In Microsoft Excel one can use the STDEV() function

	Date	Price	\$$u_j$	\$$u_j^2$
1	02-Jul-01	12140		
2	03-Jul-01	12060	-0.0066116	0.0000437
3	04-Jul-01	12060	-	-
4	05-Jul-01	11720	-0.0285974	0.0008178
5	06-Jul-01	11580	-0.0120173	0.0001444
6	09-Jul-01	11380	-0.0174220	0.0003035
7	10-Jul-01	11540	0.0139618	0.0001949
8	11-Jul-01	11720	0.0154775	0.0002396
9	12-Jul-01	12100	0.0319087	0.0010182
10	13-Jul-01	12360	0.0212600	0.0004520
11	16-Jul-01	12380	0.0016168	0.0000026
12	17-Jul-01	11740	-0.0530805	0.0028175
13	18-Jul-01	11420	-0.0276356	0.0007637
14	19-Jul-01	11460	0.0034965	0.0000122
15	20-Jul-01	11240	-0.0193839	0.0003757
16	23-Jul-01	11220	-0.0017809	0.0000032
17	24-Jul-01	11000	-0.0198026	0.0003921
18	25-Jul-01	10300	-0.0657514	0.0043232
19	26-Jul-01	10380	0.0077370	0.0000599
20	27-Jul-01	10660	0.0266175	0.0007085
21	30-Jul-01	10720	0.0056127	0.0000315
22	31-Jul-01	10800	0.0074350	0.0000553
		Mean	-0.0055695	
		StDev	0.0246051	
		Annual Vol σ	39.059401%	
			σ'	0.0259145
			Annual σ'	41.137947%

Table 1: Historical volatility for Anglo American during July 2001.

4.4 Implied Volatility

This is the volatility implied by an option price observed in the market. We illustrate the basic idea with an example. On 25 September 2001 the SASOL share priced closed at R73. Say you wanted to buy an at-the-money option that expires on 20 March 2002 (March 02 futures close-out). You phone a bank and they quote you a price of R9.00 per option[14]. From the current yield curve the riskfree interest rate is 10.5% and let's use a dividend yield of 4%. One can then substitute all of these into Equation (3) and numerically solve this to back out the volatility σ. For this example one gets an implied volatility of 40.6599%. In Appendix D I give a useful method to calculate the implied volatility.

To get reliable information on OTC option trades is difficult and there are not many listed options on the JSE. We do, however, have some warrants that are very liquid. Last year Dimension Data was the darling of the stock market and the 1DDT warrant was the most liquid listed warrant. Currently some warrants on resource stocks are very liquid. The 3AGLSB has a strike price of 11250, an expiry date of 21 February 2002 and a "warrants to share ratio"[15] of 25. In Table 2 we list the daily as well as average implied volatilities for the 3AGLSB warrant for July 2001. We also show the *Parkinson* and *Garman* & *Klass* numbers that will be discussed in Section 4.6.

4.5 Realized/Actual Volatility

This is the historical volatility calculated looking "backward" when an option expires. As an example, let's say a trader wants to write an option today, at t, that expires in 3 months time at time T. To estimate the volatility he/she might calculate the historical volatility of the past 3 months. If similar options are trading in the market he/she might calculate the implied volatility. The actual volatility will, however, only be known at expiry T. Once the 3 months have passed, one can calculate the realized volatility between t and T because the actual price path is then known.

4.6 Extreme-Value Estimators

The volatility formula in Equation (5) is called the close-close (CC) estimator. Another class of estimator (the high-low (HL)) uses intra-interval highs and lows to characterize the distribution. These estimators are more efficient because they use additional information about movements throughout the interval that snapshots at the end of an interval cannot hope to summarize. The practical importance of this improved efficiency is that five to seven times fewer observations are necessary in order to obtain the same statistical precision as the CC estimator. Since models like

[14] In SA OTC and SAFEX options are generally quoted on volatility and not on price

[15] This means that every 25 warrants gives the buyer the right to buy one share at expiry.

Date	Open	High	Low	Close	3AGL	Imp Vol	$\ln \frac{S_H}{S_L}$	$\ln \frac{S_C}{S_O}$
2-Jul	11960	12200	11920	12140	97	49.07310%	0.02322	0.01494
3-Jul	12140	12340	12060	12060	97	50.89406%	0.02295	-0.00661
4-Jul	12060	12100	11740	12060	95	49.48826%	0.03020	-
5-Jul	12000	12060	11360	11720	86	49.61197%	0.05980	-0.02361
6-Jul	11500	11680	11380	11580	80	47.88243%	0.02602	0.00693
9-Jul	11500	11700	11260	11380	74	47.52809%	0.03833	-0.01049
10-Jul	11400	11680	11180	11540	80	49.24331%	0.04375	0.01221
11-Jul	11500	11760	11400	11720	83	48.13852%	0.03109	0.01895
12-Jul	11740	12180	11740	12100	94	49.03157%	0.03679	0.03020
13-Jul	12200	12380	12080	12360	99	47.46740%	0.02453	0.01303
16-Jul	12400	12620	12300	12380	102	49.88678%	0.02568	-0.00161
17-Jul	12200	12200	11680	11740	83	48.61407%	0.04356	-0.03843
18-Jul	11700	11700	11180	11420	76	49.64077%	0.04546	-0.02422
19-Jul	11400	11500	11280	11460	76	49.00006%	0.01932	0.00525
20-Jul	11460	11460	11200	11240	70	48.65772%	0.02295	-0.01938
23-Jul	11200	11300	11040	11220	68	47.88395%	0.02328	0.00178
24-Jul	11000	11180	10960	11000	65	49.80392%	0.01987	-
25-Jul	10800	10980	10100	10300	48	48.44456%	0.08354	-0.04740
26-Jul	10360	10560	10220	10380	50	48.91137%	0.03273	0.00193
27-Jul	10400	10740	10400	10660	54	47.46794%	0.03217	0.02469
30-Jul	10700	10840	10620	10720	55	47.66888%	0.02050	0.00187
31-Jul	10540	10820	10520	10800	56	47.18901%	0.02812	0.02437
					Ave	48.7058%		
						Park	34.7877%	
							GK	36.0752%

Table 2: The Implied volatility for the 3AGLSB warrant on Anglo. Also shown are the Parkinson (Park) and Garman & Klass (GK) numbers (see Section 4.6) for Anglo American during July 2001.

Black & Scholes are based in continuous time, it is natural to want a volatility measure more broadly based on the price continuum. The pioneering work was done by *Parkinson* and *Garman* and *Klass*. We will later on show how these measures can help in the trading environment.

4.6.1 Parkinson Estimator

The form of the *Parkinson* estimator is [Pa 80]

$$\sigma_p = \sqrt{\frac{1}{n}\frac{1}{4\ln 2}\sum_{i=1}^{n}\left(\ln\frac{H_i}{L_i}\right)^2} \tag{7}$$

where H_i is the i-th interval *high* and L_i is the interval *low*. If daily data is used these will be the intra-day *high* and *low*. n is again the number of data points in the sample. σ_p is a general average over n intervals for the intra-interval volatility.

4.6.2 Garman-Klass Estimator

Garman and *Klass* improved the efficiency by using even more information [GK 80]. They also used the intra-interval *open* O_i and *close* C_i prices to obtain

$$\sigma_{gk} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{2}\left(\ln\frac{H_i}{L_i}\right)^2 - (2\ln 2 - 1)\left(\ln\frac{C_i}{O_i}\right)^2\right]} \tag{8}$$

To include overnight volatility one can substitute the opening price O_i with the previous interval's closing price C_{i-1}.

Table 2 list the values of the *Parkinson* and *Garman* and *Klas* estimators for the same data used in Table 1. We see that the intraday volatility is much lower than the historical and implied volatilities.

5 What Volatility?

Though there are no survey results to support the following claim, there is reason to believe that if 10 different quantitative analysts were asked to supply the volatility of a security, at least nine different answers would be offered [Ne 97]. Does this means that quants are careless hacks practising careless, ad hoc methods? Or, is the volatility measure defined haphazardly?

Neither one is true. The disparity comes largely from the multitude of ways one could sample the data in calculating the standard deviation. Some traders with a long memory may argue that a longer sample, like a year, is necessary because more observations lead to better statistical accuracy. Other traders may be victims of some form of market amnesia and may favour a relative short sample period, like a month, under the assumptions that the recent past is more indicative of the current environment. Market shocks (like the influence the attacks on the World Trade Centers on 11 September 2001 had on the market) influence these traders' view of the market and they give this data a high weighting in their volatility analysis. Others argue that one needs a sample based on the length of the time to maturity i.e., if the option expires in four months time, one needs to have a sample of four months of data to calculate the volatility.

Another source of disparity comes from the return (as defined in Equation (4)) calculation interval. Should the time subscript i be measuring weeks, days or even higher frequency intraday periods? Yet another way to characterize the width of the return distribution is to use the intraday highs and lows. We have described this in Section 4.6.

Still others may contend that the historical volatilities are unimportant because the market has already gleaned all the information from past movement and has incorporated the information into option prices. If the market is efficient at using all available information in forming the option price, and if the price model is correct, then the implied volatility is the best guess possible for ensuing volatility.

With so many methodological variations available, it seems impossible to specify the "right" approach to estimation. Indeed, there are *no universal truths*. There are, however, conditional truths and guidelines that will be discussed in the following sections.

6 Volatility is not Constant

We have mentioned at the start that as far back as 1976, *Fischer Black* stated that volatility is not constant. Figure 3 through 6 will show the trader to beware the notion of constant volatility. Figure 3 shows the implied volatility for the 1AGL warrant from July 1999 to September 2001. Figure 4 shows the one month historical volatility for the Alsi100 from June 1987 to September 2001 using daily data. See how the volatility shot up during the crashes of October 1987 and September 1997, again during the Asian crises and after the World Trade Center attacks.

Measuring the volatility of volatility shows the volatility to be even more volatile than the underlying security itself [Ta 97]. When calculating the volatility of volatility, one should be careful to avoid running overlapping data. In addition, the length of the period matters, and it is recommended to select as short a period between sampling points as possible. Figure 5 shows the volatility of historical volatility for the Alsi100 in nonoverlapping periods from May 1988 and Figure 6 depicts the 2-week historical volatility of the implied volatility for the 1AGL warrant.

7 Sample Size

In Section 5 we pointed out that the sample size in calculating the historical volatility is a contentious issue. We noted there are no fixed rules but we discuss two ways to get a better feel for what sample size might be more appropriate.

7.1 Volatility Plots

A useful tool is a plot of volatilities with different window-lengths (intervals). For instance, moving windows of one-month volatilities could be plotted with moving windows of three-month volatilities superimposed. See Figure 7 where we plotted the 1 month, 3 month and 12 month historical volatilities for the Indi25 from June 1995.

This can show how stable volatilities have been throughout history. It can also jog memories of specific events that may have caused spikes in volatility. If the trader does not believe that a similar event will happen over the life of the option, the best volatility estimate will exclude that point.

Another property this tool can reveal is the effect of outliers on the volatility estimation. Certainly an event like 19 October 1987, shows up as a large volatility plateau on a plot like this. In Figure 7 one can clearly see the crash of September 1997 and a few months later the Asian crises during 1998. The 11 September 2001 World Trade Center attacks is just starting to show on the left of the graph. Not only is there a big jump when the point enters the sample, but also a precipitous drop as soon as it leaves the sample. The drop-off is bothersome. Does the market perceive risk so differently from the one day to the next? Probably not[16].

To overcome this one can use a time-weighted volatility.

7.2 Introducing Filtering

One way to handle the drop-off eluded to in the previous section is to filter the data. Filtering is a simple method for taking into account that events in the past need to have uneven weightings. Here we present a simplified version of the *Kalman* filter[17]: the exponential decay.

It gives the trader the flexibility to assign more importance to recent events in proportion to their distance away from the present. Note, however, that the trader should not take the measurement for gospel. He might have information about some political situation or some structural changes took place that need to have an effect on the weighting.

The symbol λ is used for the decay factor where $0 \geq \lambda < 1$. To lengthen the memory let λ be closer to one. In using the noncentered volatility defined in Equation 6 we obtain the filtered volatility as

$$\sigma'' = \sqrt{\frac{1}{\Omega}\sum_{j=1}^{n}\lambda^{n-(j-1)}u_j^2} \qquad (9)$$

[16] Implied volatilities are not likely to behave that way

[17] For an introduction see http://www.computing.edu.au/ robin/thesis/node15.html.

	Date	Index	u_j	u_j^2	λ^j	$\lambda^j u_j^2$
1	02/07/2001	8885.5				
2	03/07/2001	8766.7	-0.0134603	0.00018118	0.34056	0.00006170
3	04/07/2001	8596.2	-0.0196402	0.00038574	0.35849	0.00013828
4	05/07/2001	8489.5	-0.0124901	0.00015600	0.37735	0.00005887
5	06/07/2001	8430.6	-0.0069622	0.00004847	0.39721	0.00001925
6	09/07/2001	8412.1	-0.0021968	0.00000483	0.41812	0.00000202
7	10/07/2001	8506.5	0.0111594	0.00012453	0.44013	0.00005481
8	11/07/2001	8475.0	-0.0037099	0.00001376	0.46329	0.00000638
9	12/07/2001	8546.6	0.0084129	0.00007078	0.48767	0.00003452
10	13/07/2001	8606.4	0.0069726	0.00004862	0.51334	0.00002496
11	16/07/2001	8582.9	-0.0027343	0.00000748	0.54036	0.00000404
12	17/07/2001	8400.5	-0.0214806	0.00046142	0.56880	0.00026245
13	18/07/2001	8276.7	-0.0148469	0.00022043	0.59874	0.00013198
14	19/07/2001	8410.1	0.015989	0.00025565	0.63025	0.00016112
15	20/07/2001	8313.8	-0.0115166	0.00013263	0.66342	0.00008799
16	23/07/2001	8280.6	-0.0040014	0.00001601	0.69834	0.00001118
17	24/07/2001	8153.0	-0.0155295	0.00024116	0.73509	0.00017728
18	25/07/2001	8063.7	-0.0110134	0.00012130	0.77378	0.00009386
19	26/07/2001	8002.1	-0.0076685	0.00005881	0.81451	0.00004790
20	27/07/2001	8032.4	0.0037794	0.00001428	0.85738	0.00001225
21	30/07/2001	8120.1	0.0108591	0.00011792	0.90250	0.00010642
22	31/07/2001	8124.7	0.000566335	0.00000032	0.95000	0.00000030
				SUM	12.52933	0.00149756
					σ''	17.355144%
					σ'	17.937605%

Table 3: Filtered volatility for Alsi40 with $\lambda = 0.95$.

where

$$\Omega = \sum_{j=1}^{n} \lambda^j = \lambda + \lambda^2 + \lambda^3 + \ldots + \lambda^n \approx \frac{1}{1-\lambda}$$

For the last approximation to hold, n needs to be very large (higher than 1000 observations as a general rule).

The closer λ is to 1, the longer the memory included in the calculation where $\lim_{\lambda \to 1} \sigma'' = \sigma'$. Table 3 list the values of the filtered volatility for the Alsi40 with $\lambda = 0.95$. We calculate the filtered volatility to be 17.355% (σ'' scaled by $\sqrt{252}$) while the noncentered volatility σ' is 17.938%. *Van Vuuren*, *Botha* and *Styger* mentions that, in general, emerging markets are assigned a $\lambda = 0.94$. They analyzed the South African market and give guidelines in estimating λ on a daily basis which they found to lead to more accurate volatility and hence Value-at-Risk values [VB 00].

8 The Parkinson Number and the Variance Ratio Method

An important use of the Parkinson number as defined in Equation (7) is the assessment of the distribution or prices during the day as well as a better understanding of market dynamics. Comparing the *Parkinson* number and the periodically sampled volatility helps traders understand the mean reversion in the market as well as the distribution of stop-losses. Some clear rules can be derived from that information.

Comparing the *Parkinson* number σ_p with the definition of periodically sampled historical volatility σ' gives

$$\sigma_p \simeq \sqrt{4 \ln 2}\ \sigma' \approx 1.67\sigma' \tag{10}$$

This means that the volatility of the market as observed through the 24 hours or one week, or any stable sampling period should be related to the volatility as measured by the extremes[18]

The relation in Equation 10 can give meaningful information with the following situations:

- **Pricing Barrier Options**: they get triggered through an actual trade happening during trading hours; therefore, the distribution of the extremes is most important. The barrier option trader needs only to know the high or the low to see if the option was knocked-in or knocked-out. How this extreme is distributed matters more than the close-to-close volatility. The *Parkinson* number is the sole information. If there is a bias making the σ_p consistently higher than $1.67\sigma'$, the trader knows that the probability of hitting the trigger is higher.

- **General Option Delta Adjustment**: Equation (10) can reveal information about the mean reversion of a particular market. This allows the trader to set his frequency of hedging accordingly. If σ_p is higher than $1.67\sigma'$ the trader needs to hedge a long gamma position more frequently. Otherwise he could lag the needed adjustment, a technique called "letting the gammas run".

- **General trading Strategies**: the market maker's edge is strongest in cases where σ_p is higher than $1.67\sigma'$. It is otherwise better to follow a trend.

Figure 8 shows the *Parkinson* number ratio to volatility for RCH from December 1988 for one month volatilities. From this we conclude that a trader does not need to hedge RCH options as often as the option greeks show.

Another test for mean reversion is done by looking at hourly and daily volatilities. This is called the variance method. *Lo* and *MacKinley* showed that if the volatility on

[18]Warning: such measurement cannot be used to compare close-to-close volatility with intraday high/low. It can compare 24-hour high/low to data sampled every day at the same time. For equities that only trades during the day, it is better to use open-to-close volatility.

an hourly sampled basis turns out to be higher than the volatility on a daily sampled basis, the market can be considered as mean reverting [LM 88]. On the other hand, if the market showed a higher volatility at a wider frequency between the dates, then it can be concluded that there is a trend. Figure 9 shows the volatility sampled at different frequencies for the Indi25 during June 2001. From this we conclude that the market was mean reverting. The market just moved sideways where it started at 7556 on 1 June and ended at 7583 on 29 June.

9 High Frequency Data

Today, an enormous amount of data is available in the financial markets. People are starting to utilize this information to get a better understanding of the dynamics of the market intraday i.e., during trading hours.

Trading activity is not uniform during trading hours, either in terms of volume or in number of contracts. Rather, a daily cycle is observed in market data [MS 00]: the volatility is higher at the opening and closing hours, and usually the lowest value of the day occurs during the middle hours. As an example we show in Figure 10 the intraday 2-hour volatility using 2 minute reading for the Alsi40 from 13 September 2001 to 26 September 2001. These dates include the date when the markets in the USA opened after the attack on the World Trade Centers in New York. The jump in the market on Tuesday 25 September is reflected in the huge volatility shown to the right of the graph. Clearly visible is a daily cycle where the volatility is the highest at the opening. Just to show that this is not a fluke of the extraordinary circumstances of 11 September, we show in Figure 11 the intraday volatility from 20 March 2001 to 30 March 2001. The same pattern is visible. The same pattern holds for markets across the globe.

10 Conclusion

The subject of volatility is vast. In this paper we first introduced the *Black & Scholes* model and showed the importance they put on the volatility parameter. Although some parts of this paper is rather theoretical, we concentrated on the practical uses of these concepts. We stressed the importance of volatility and the estimation thereof. We have defined various ways in estimating the volatility and have given guidelines on how these can be used to obtain useful information that will help the trader and risk manager. From the analysis we showed that volatility is not constant and we have described some practical methods that will help practitioners to understand the dynamics of volatility.

Appendix

A A Word on Modeling

In analyzing the world around us mathematically (whether it be atoms or economic systems), models have to be created. These models capture as many aspects as possible of the real system. In order to obtain models that can be analyzed mathematical rigorously, some simplifications will be necessary. Even then, most systems are still too complex to analyze exactly. These models usually take the form of differential equations that have to be solved to obtain physical answers.

Under uncertainty the situation is even more complex. The construction of models requires that we distinguish known from unknown realities and find some mechanisms to reconcile our knowledge with the lack of it. For this reason modeling is not merely a collection of techniques but an art in blending the relevant aspects of a problem and its unforeseen consequences with a descriptive, yet tractable, mathematical methodology. To model under uncertainty, we typically use probability distributions to describe quantitatively the set of possible events that may unfold over time [Ta 88]. From this, stochastic differential equations can be obtained. Specification of the structure of the probability distributions are important and based on an understanding of the process. Moments of such processes (particularly the mean and variance) tend to reflect the trend and the degrees to which we are more or less certain about events as they occur.

Always remember that a model is not reality, *but something that imitates reality at a certain scale* [Mo 91].

B The South African Options Market

In South Africa option prices are quoted on volatility and not price. This means that the quote may be 30/32 for a particular option. The writer is thus willing to sell the option at a volatility of 32% and buy the option at 30% volatility. It seams that one does not have to calculate the implied volatility; it is supplied by the market. The question may still be, is the quoted volatility correct or are there other market forces that drives the volatility?

In South Africa, though, we do not have a very liquid options market. The Safex index futures option market is not very deep. We only have a few listed options on the JSE and most single sock options are OTC trades and this is only now starting to get off the ground. The warrant market is growing but it is still small compared to the underlying market. Hugh discrepancies can occur due to supply and demand.

Because the South African option market is not very efficient, implied volatility might not always give an accurate reflection of the volatility of the underlying security.

C Solution to the Black & Scholes Differential Equation

Black & Scholes solved the partial differential equation (PDE) given in (2) for a stock that does not pay a dividend. We here follow their methodology in solving the call option and put $d = 0$. The boundary condition for a call is given in Eq. (1). Subsequently researchers have developed other methods to solve this PDE and other methods to solve the option pricing problem without having to solve the PDE directly e.g., using probability theory [Hu 97, HK 00].

Fischer Black was a physicist and in mentioning the problem to his colleagues they realised that this rather "complex-looking" PDE could be transformed into a form that looks simpler and was known. They made the following ansatz [BS 73]

$$V(S,t) = e^{-r\tau} y(u,v) \tag{11}$$

where

$$\begin{aligned} u(S,t) &= \frac{2}{\sigma^2}\left(r - \frac{\sigma^2}{2}\right)\left[\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)\tau\right] \\ v(S,t) &= \frac{2}{\sigma^2}\left(r - \frac{\sigma^2}{2}\right)^2 \tau. \end{aligned}$$

This substitution transforms (rotates) the *Black & Scholes* PDE to

$$\frac{\partial y}{\partial v} = \frac{\partial^2 y}{\partial u^2} \tag{12}$$

and the boundary condition becomes

$$y(u,0) = \begin{cases} 0, & u < 0 \\ K\left[\exp\left(\dfrac{u\sigma^2/2}{r - \sigma^2/2}\right) - 1\right], & u \geq 0. \end{cases}$$

Equation (12) is the PDE for heat transfer through a medium; well-known to the physics community [Ha 87, Bo 83]. This equation has been studied thoroughly by physicists and the solution (using these boundary conditions) is obtained by using Fourier series (see [Ch 63] or any good book on this subject).

In our notation the solution is given by

$$y(u,v) = \frac{1}{2\pi}\int_{-u/\sqrt{2v}}^{\infty} K\left(\exp\left[\frac{(u + q\sqrt{2v})\sigma^2/2}{r - \sigma^2/2}\right] - 1\right) e^{-q^2/2} dq. \tag{13}$$

Substituting (13) into (11), and simplifying we find

$$V(S,t) = S(t)N(x) - Ke^{-r\tau}N(y) \tag{14}$$

where $N(x)$ is the Normal/Gaussian distribution function given by

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-a^2/2} da$$

and

$$\begin{aligned} x &= \frac{1}{\sigma\sqrt{\tau}} \left[\ln\left(\frac{S}{K}\right) + (r + \frac{1}{2}\sigma^2)\tau \right] \\ y &= \frac{1}{\sigma\sqrt{\tau}} \left[\ln\left(\frac{S}{K}\right) + (r - \frac{1}{2}\sigma^2)\tau \right] = x - \sigma\sqrt{\tau}. \end{aligned}$$

Equation (14) is the well-known *Black & Scholes* solution for the value of a call option. The put value can be obtained by either imposing the put boundary conditions onto (2) and solving or from the *put-call-parity* relation $C + Ke^{-r\tau} = P + S$ such that

$$V(S,t) = Ke^{-r\tau} N(-y) - SN(-x). \tag{15}$$

Note that the expected return on the stock does not appear in the solutions and the solutions are risk-neutral.

D Calculating the Implied Volatility

The "brute force" technique is to use a numerical routine to solve Equation (3) for σ. This can be done quite easily by using a Newton-Raphson technique [PF 86].

Corrado and *Miller*, however, derived an accurate formula to compute implied volatility [Ne 97]. They refer to it as the improved quadratic formula where

$$\sigma\sqrt{T} = \frac{\sqrt{2\pi}}{S+X} \left[V - \frac{S-X}{2} + \sqrt{\left(V - \frac{S-X}{2}\right)^2 - \frac{(S-X)^2}{\pi}} \right].$$

Here $X = Ke^{-rT}$ is the discounted strike price, S is the stock price and T is the time to expiry. It is accurate over a wide range of strike prices.

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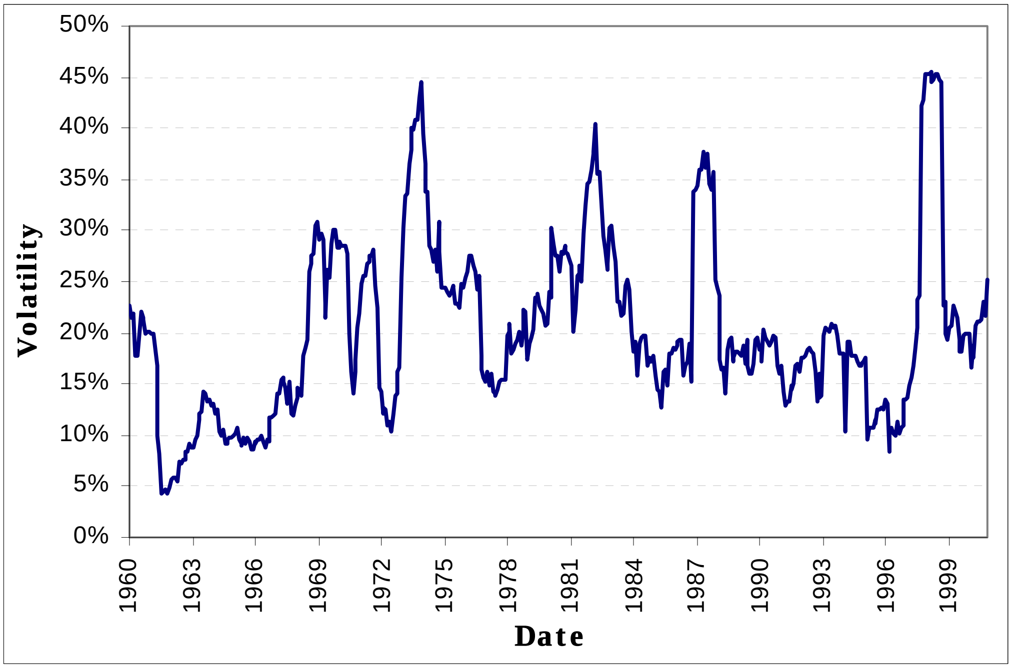

Figure 1: Monthly volatility of the JSE All Share Index since 1960

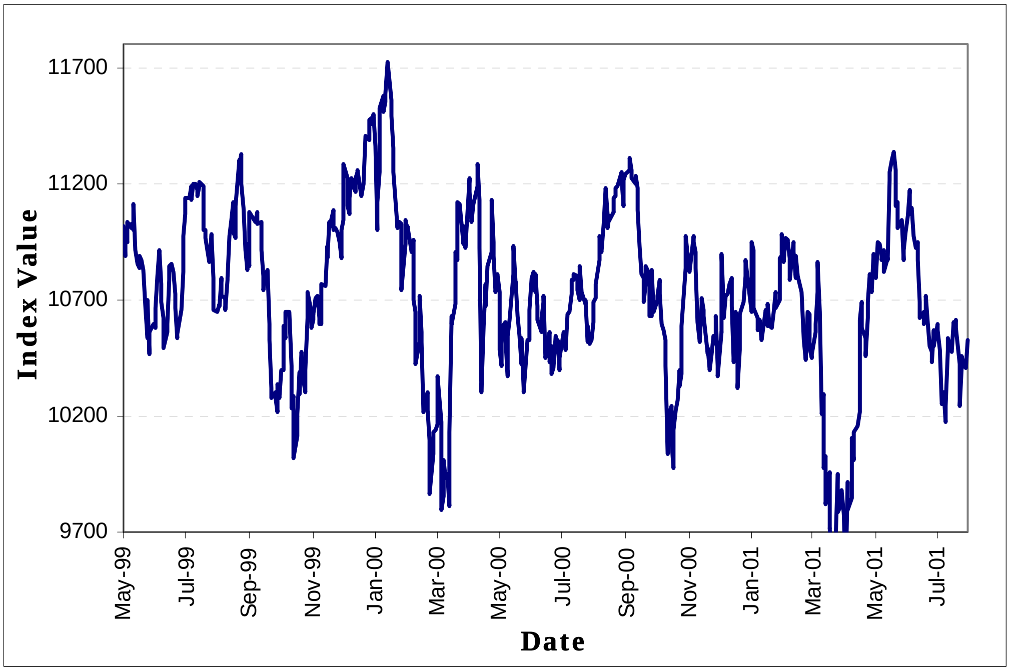

Figure 2: The Dow Jones Industrial Average from May 1999 to July 2001

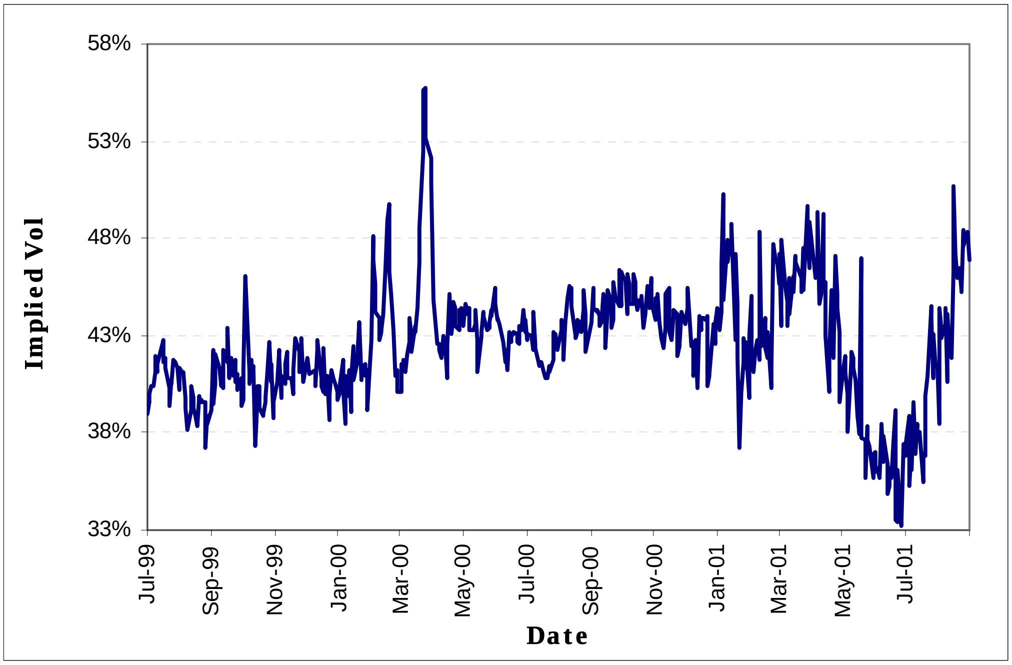

Figure 3: Implied volatility of the 1AGL warrant

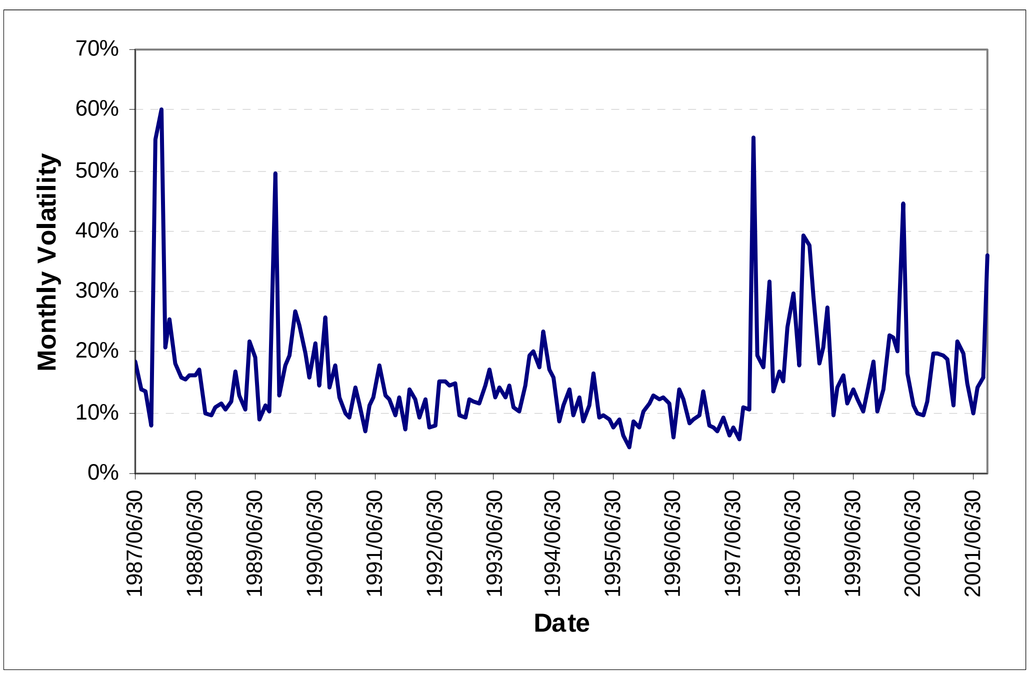

Figure 4: One month historical volatility for Alsi100

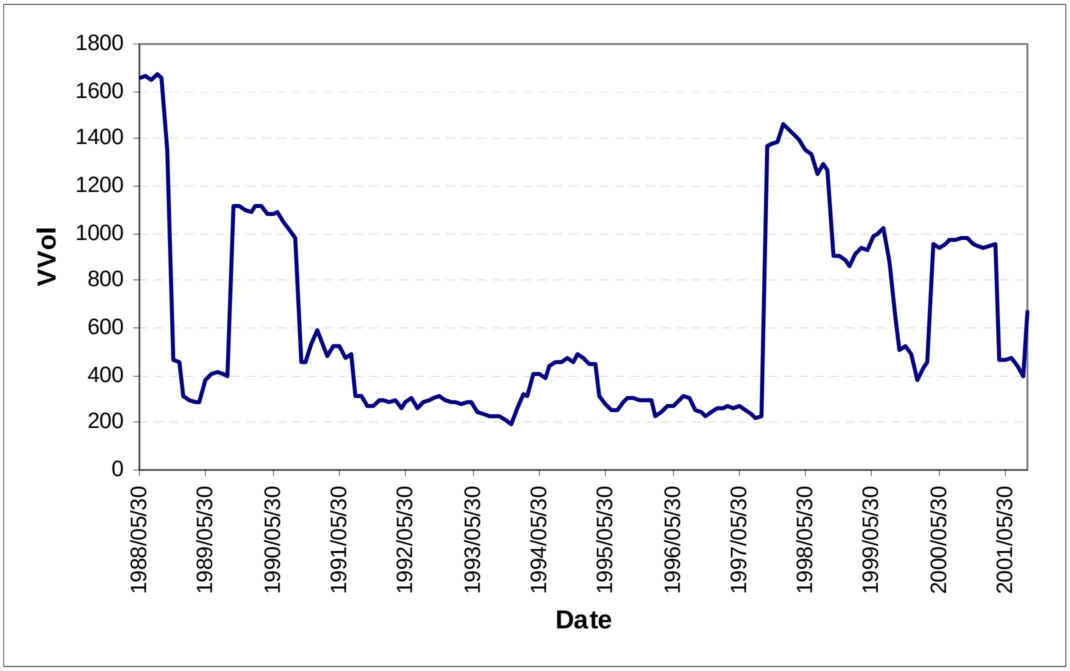

Figure 5: Volatility of the one month historical volatility for Alsi100

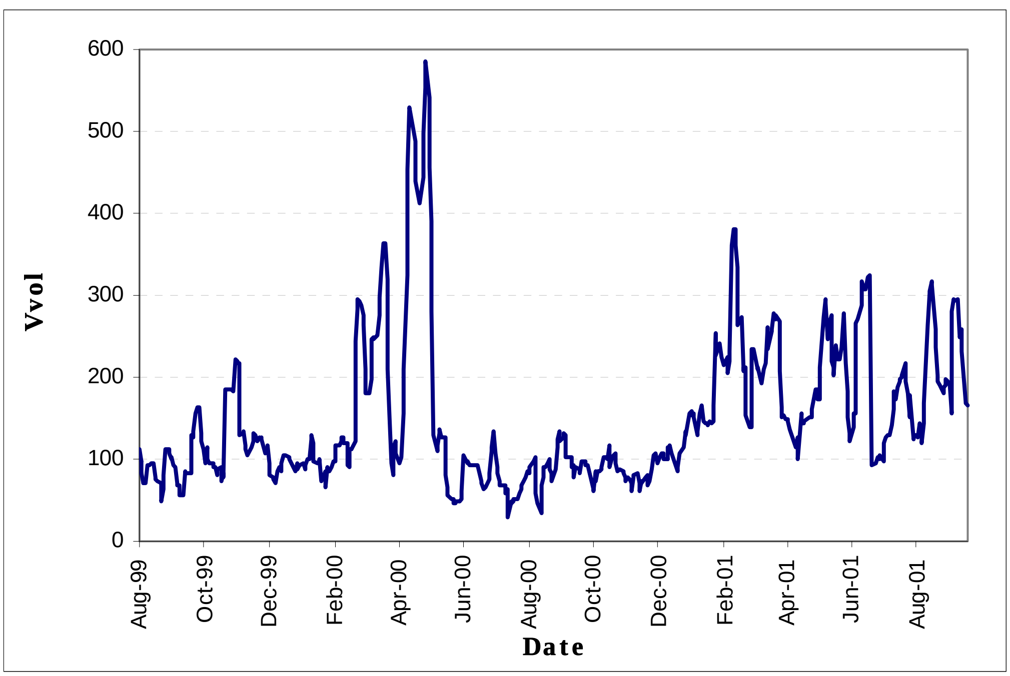

Figure 6: Two week historical volatility of the 1AGL implied volatility

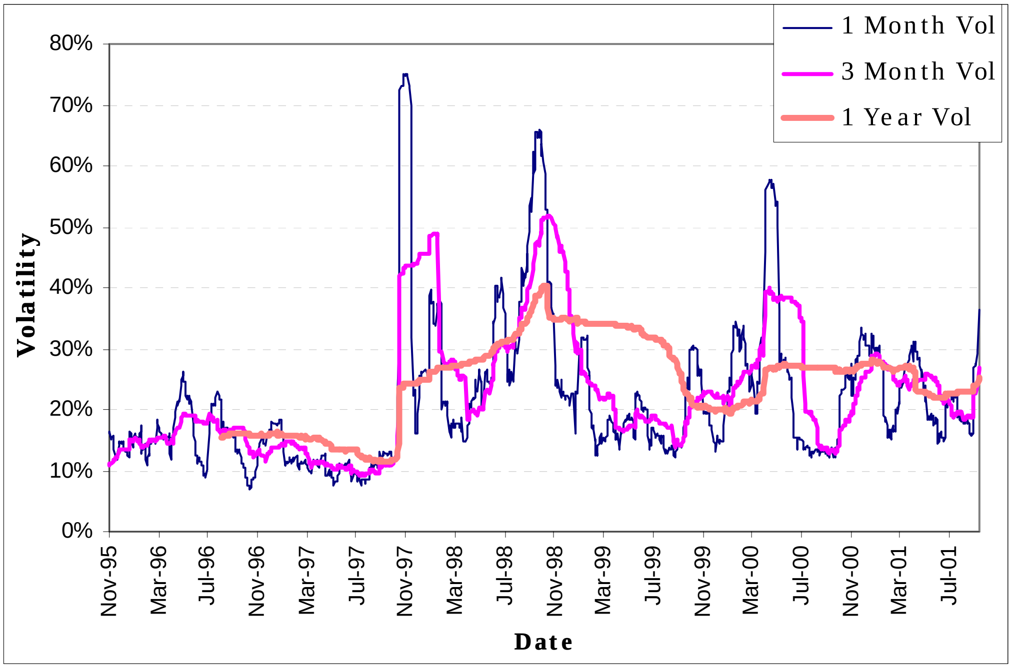

Figure 7: Indi25 moving windows of volatility

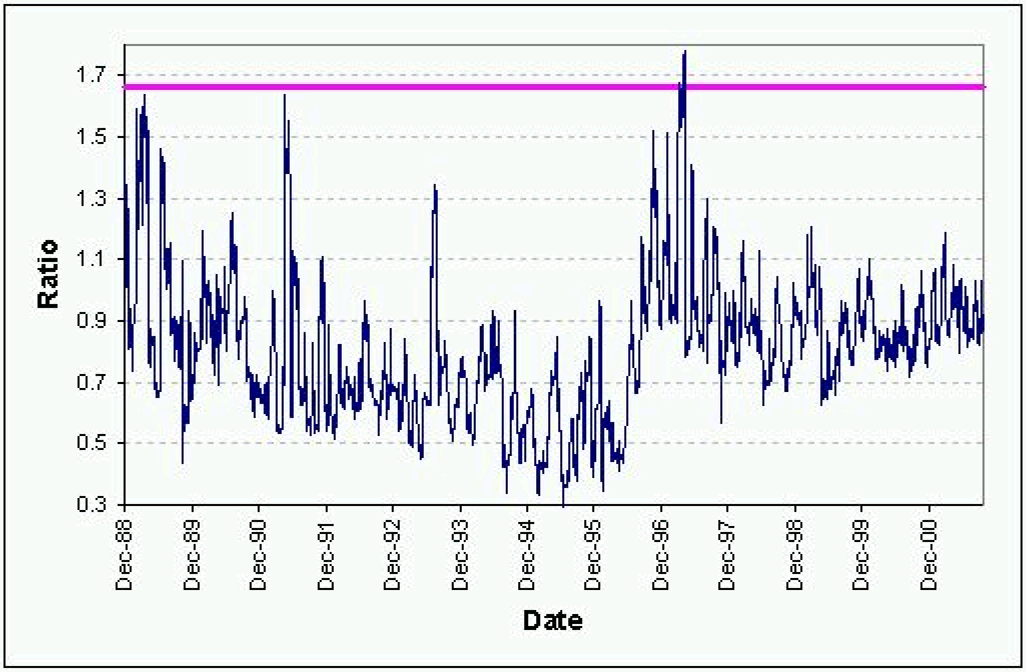

Figure 8: Ratio of the Parkinson number to historical volatility for RCH

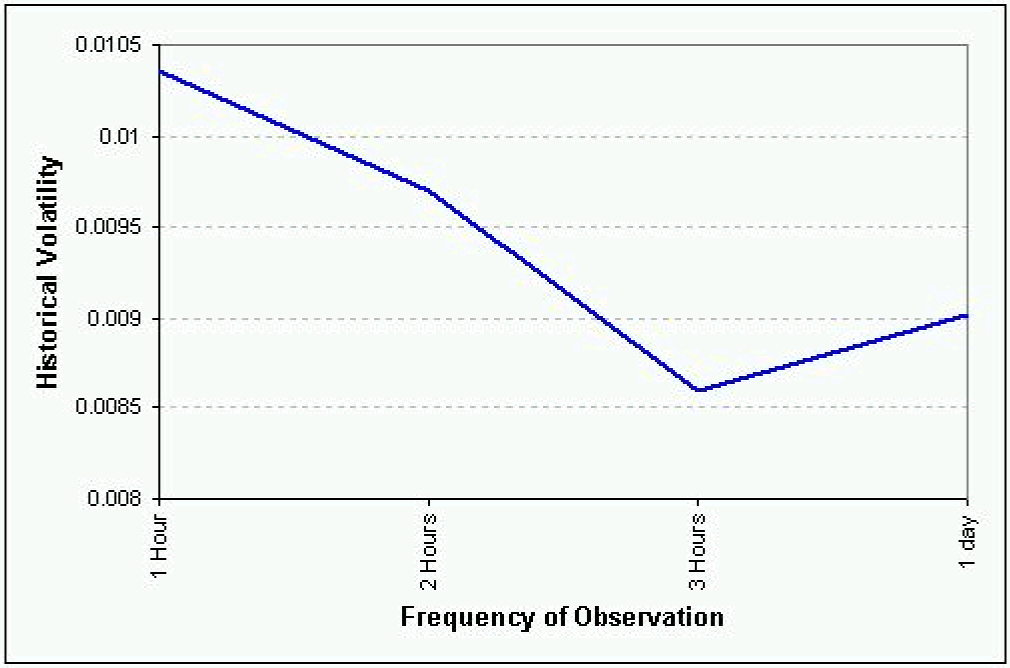

Figure 9: Variance ratio method for Indi25 during June 2001

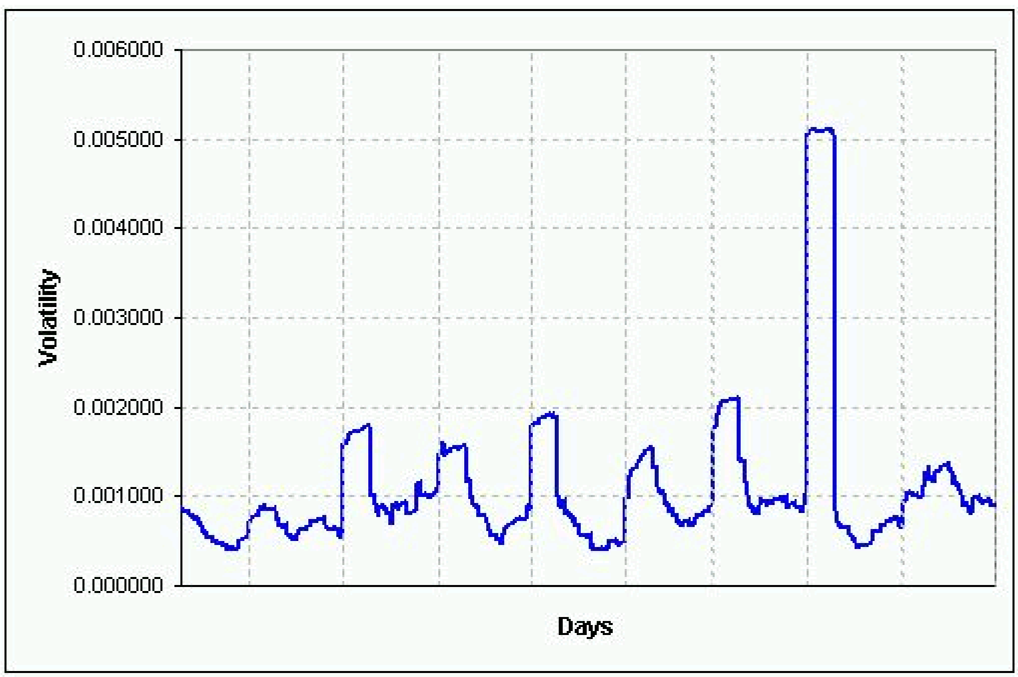

Figure 10: A daily cycle is visible in the high frequency time evolution of the Alsi40. Here we show the 2 hourly volatility calculated from 13 September 2001 to 26 September 2001.

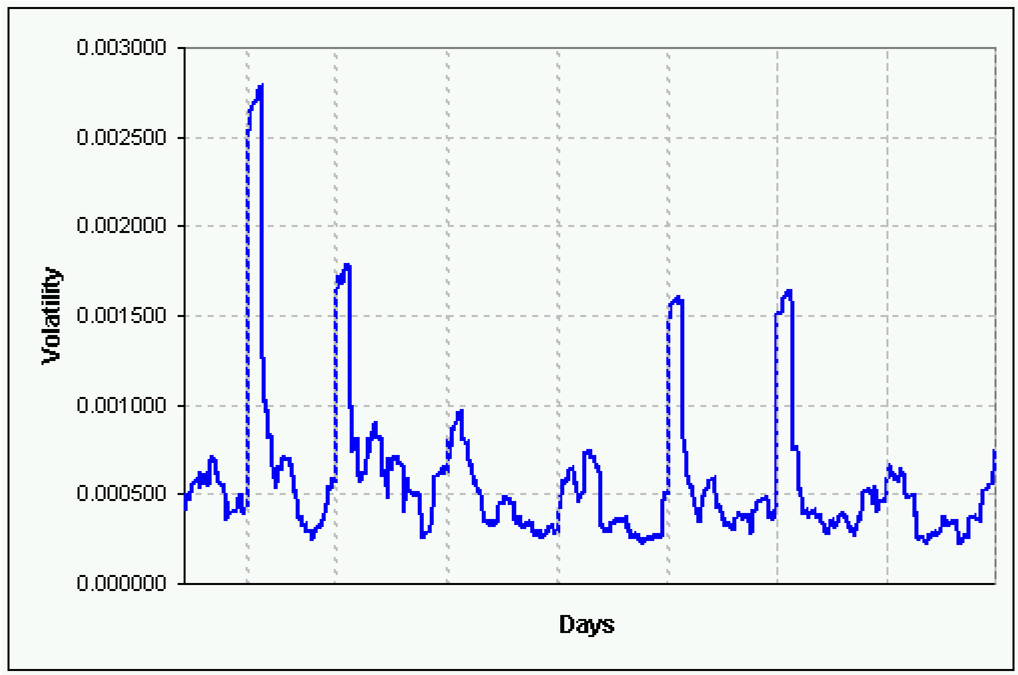

Figure 11: The same as for Figure 10 but for data from 20 March 2001 to 30 March 2001. The same pattern is visible.